Entrenchment and Investment

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May 16, 2012

ABSTRACT

This paper shows that restrictions on the issuance of non-voting shares may cause managers who own equity in the firm to under-invest. When a firm issues voting shares to raise capital for new investment, there is a dilution in the manager’s ownership. This increases the risk to the manager’s control of the firm, decreasing his chances of obtaining the private benefits of control. The problem is most severe in firms where managers extract significant private benefits. Non-voting stock allows a firm to raise equity capital without a dilution in the manager’s ownership and alleviates the under-investment problem. There are costs to the issuance of non-voting stock – managerial entrenchment, dividend dilution and firms in the control of inferior managers. The issuance of non-voting equity is optimal when the benefits, higher firm value because of higher investment outweighs the cost of managerial entrenchment and dividend dilution. We obtain conditions under which it is optimal for firms to issue non-voting stock. Our theory is consistent with the empirical findings of Faccio and Masulis (2005) who show that a fear of loss of control makes shareholders reluctant to issue voting equity to finance M&A activities. In addition, our model produces new empirical predictions regarding the relationship between the likelihood of dual-class recapitalization and incumbent management quality, management ownership and the effectiveness of other mechanisms to restrict private benefits.

Keywords: Blockholders, Controlling shareholders, Dual-class, Hostile takeovers, Ownership structure, Private benefits of control, Shareholders’ welfare, Takeover defenses, Underinvestment, Voting rights.

JEL Classification Code: G32, G34, G38, K20

*Corresponding author’s tel: +61293855860 and email: ron.masulis@unsw.edu.au. Suman Banerjee (sbanerjee@ntu.edu.sg) gratefully acknowledges financial support (SUG Tier 1) from the Ministry of Education, Singapore. We would like to thank Harry DeAngelo, David Denis, Francois Derrien, David Hirshleifer, Paolo Fulghieri, Clive Lennox, Bill Megginson, Stewart Myers, Thomas H. Noe, Raghuram Rajan, Matthew Spiegel, Neal Stoughton, Josef Zechner and the seminar participants at the Cornerstone Research, Hong Kong University of Science & Technology, Nanyang Technological University, National University of Singapore for their insightful comments. All remaining errors are our own.
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This paper shows that restrictions on the issuance of non-voting shares may cause managers who own equity in the firm to under-invest. When a firm issues voting shares to raise capital for new investment, there is a dilution in the manager’s ownership. This increases the risk to the manager’s control of the firm, decreasing his chances of obtaining the private benefits of control. The problem is most severe in firms where managers extract significant private benefits. Non-voting stock allows a firm to raise equity capital without a dilution in the manager’s ownership and alleviates the under-investment problem. There are costs to the issuance of non-voting stock – managerial entrenchment, dividend dilution and firms in the control of inferior managers. The issuance of non-voting equity is optimal when the benefits, higher firm value because of higher investment, outweigh the costs of managerial entrenchment. We obtain conditions under which it is optimal for firms to issue non-voting stock. Our theory is consistent with the empirical findings of Faccio and Masulis (2005) who show that a fear of loss of control makes shareholders reluctant to issue voting equity to finance M&A activities. In addition, our model produces new empirical predictions regarding the relationship between the likelihood of dual-class recapitalization and incumbent management quality, management ownership and the effectiveness of other mechanisms to restrict private benefits.

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1 Introduction

In the movement toward shareholder democracy, equalizing the voting power and influence of common shares has become a touchstone for good governance. Corporate charter provisions that explicitly limit the rights of minority shareholders are completely antithetical to this movement. Thus, it is not surprising that dual-class provisions, which create a second class of common stock with reduced voting power, have come under fire.

More generally, activist shareholders seem concerned about disenfranchisement of holders of inferior classes of shares. Institutional Shareholder Services (ISS) recently recommended that dual-class share structures be eliminated entirely for all future newly listed companies. Also, ISS wants corporate laws to be changed to require sunset provisions for companies with this kind of structure, such that all shares will revert to common shares after some time, unless a majority of inferior-class shareholders vote to reconfirm the dual-class structure. The corporate democracy movement has led policy makers, most vocal among them a high-level group of EU company law experts and Indian corporate activists, to warn of the threats posed by dual-class provisions.

Theoretical literature on dual-class shares has analyzed non-voting equity in the context of control contests and finds that dual-class ownership structures produce a negative shareholder wealth effect (e.g., Grossman and Hart (1988), Harris and Raviv (1988) and Ruback (1988)). These papers trace this negative wealth effect to the unbundling of voting rights and cash flow rights—arguing that the unbundled votes can act as an anti-takeover device. When the likelihood of a successful value increasing takeover is diminished, it changes managerial incentives by allowing the managers to deviate from making financing and investment decisions that enhance shareholder’s wealth. In addition, the market for corporate control weakens as a disciplining mechanism, providing the incumbents greater latitude to consume private benefits.

We argue for a more nuanced view of the role of dual-class shares. We demonstrate that although a dual-class share structure weakens the incentives associated with the market for corporate control, it helps to mitigate the problem of under-investment resulting from the noncontractability of the firm’s investment policy. That is, on the one hand, for a firm with

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1Surprisingly, dual-class structures are very common all over the globe: Many prominent and highly valued companies such as Berkshire Hathaway Inc., Blackstone Group, Clearwire, Dolby, Echostar Communications, Ford, Fox Entertainment Group, Google, MasterCard, Rosetta Stone, VISA, VMWare, and WebMD. Another well-known example is News Corp. Dual-class structures are widely used in seven European countries: Denmark, Finland, Germany, Italy, Norway, Sweden, and Switzerland. Dual-class structures are common in Canada, where 5 to 6% of listed companies have them; examples include Metro, Bombardier, Gluskin Sheff, Air Canada, Exfo, Cossette, and Celestica.

2According to the EU company law experts’ recommendations, a bidder that has acquired 75% of the risk based capital of a company should be able to gain control, even if it acquires shares with inferior or no voting rights.

3See, for example, Shleifer and Wolfenzon (2002) who show firms with weaker shareholder protection have lower valuations because investors take into account that some of the profits can be diverted.

4According to the Library of Canadian Parliament, “Undeniably, some of the best-performing companies in Canada have multiple-voting share structures. Thus, not all shareholders are concerned with the
dual-class hare structure, scale-expanding investment projects generally require sizable new equity, which in turn reduces incumbent management ability to resist takeovers. Thus, rational managers with noncontractable control benefits have an incentive to reject such scale-expanding investments. On the other hand, Dual class shares protect managers from the loss of control rents, and in turn encourage value maximizing investment policies. Particularly for smaller firms facing large, profitable investment opportunities, a dual-class share ownership structure can increase shareholder wealth and the firms overall economic efficiency.

In essence, we propose that incumbent managers face a clear trade-off when choosing what type of common shares to issue to fund new investment projects: dilution of their control rights versus dilution of their cash flow rights. For example, an incumbent manager who owns some voting shares in the firm and extracts control benefits, if forced to use voting shares to fund new investment projects, may prefer to forgo some positive NPV projects: Because issuing voting shares to outside shareholders dilutes the incumbent’s effective control, it thereby increases the likelihood that the incumbent will lose control of the firm if he chooses a sufficiently high level of investment. Along these lines, Faccio and Masulis (2005) contend that a bidder’s financing choice critically depends on its ownership structure. They find that the corporate control incentives to use nonvoting shares to fund a project are likely to be strongest when a target’s share ownership is concentrated and a bidder’s largest shareholder has an intermediate level of voting power—that is, in a range where he is most vulnerable to a loss of control under a stock-financed acquisition.

At the same time, however, an incumbent who is allowed to use nonvoting shares will not necessarily use them to finance new investments, because the issuance of nonvoting shares also comes at a cost. Though the strategy allows the incumbent to maintain effective control, it also dilutes the cash flow rights of all existing shareholders— including the incumbent— as more nonvoting shares relative to voting shares must be issued to raise same amount of investible funds. But an incumbent who is forced to use voting shares to finance new investments will not necessarily underinvest, either. If the incumbent owns a significant portion of the total cash flow rights, he bears a significant portion of the loss due to under-investment, and may choose

voting rights attached to a share. They may be more interested in the potential of sharing the company’s wealth or trading on future prospects by buying cheaper, subordinated shares.” Read more at http://www2.parl.gc.ca/Content/LOP/ResearchPublications/prb0526-e.htm.

For example, large private placements of voting shares can endanger the effective control of the incumbent by creating a contesting outside block. This may lead the incumbent to forgo the investment that he intended to fund with the proceeds from the private sale to protect his control rights or forgo the equity issue itself. Thus, governance quality and ownership structure become important determinants of the decision to raise investible funds using private placements. See, for example, Wruck (1989) and Wruck and Wu (2009) for detailed discussions.

Some dual-class firms are created to favor Canadian ownership in strategic or culturally sensitive fields. A dual-class structure allows Canadians to control the votes, while allowing equity to be raised and held on an international basis. Many foreign investors have happily bought into structures of this sort,” said Barry Reiter of Bennett Jones LLP.

For example, Wruck (1989) finds that the average management-controlled holdings fall by 1.5% around the time of the private equity sale.
to dilute his control rights to reap the benefits of additional investments.

To add yet another wrinkle to the decision process, we observe that outside shareholders, who are assumed to have approval rights, also face a trade-off when considering share issue for an investment project: higher managerial entrenchment versus more positive NPV investments. Outside shareholders know that if they allow the incumbent manager to issue both voting and nonvoting shares, then the incumbent will invest in all available positive NPV projects. But outside shareholders also know that when this additional investment is funded with nonvoting shares, they bear another cost: The incumbent manager is relatively more entrenched because his private benefits play an enhanced role in the control contest. Hence, future takeover activities may be adversely affected resulting in lower expected takeover premiums. However, if outside shareholders force the incumbent to use only voting shares, then he may forgo some positive NPV projects, which is also costly for the outside shareholders. For example, DeAngelo and DeAngelo (1985) indicate that there can be an under-investment problem, if the managers are faced with increasing the risk of losing control when they fund new projects with voting equity.8

Taking into considerations all of the possibilities, we propose that differences in investment opportunity sets may help to explain the considerable variation in the effects of dual-class share issues both within and between firms. Others have shown that deviations from "one share-one vote" can be optimal, but ours is the first to integrate the dual-class decision (heretofore viewed simply as of concern in the control literature) into the rich body of research on capital structure and under-investment. We focus specifically on the firm’s decision to forgo positive NPV investment opportunities.9

In a corporate control context, the problem of under-investment differs considerably from standard under-investment scenarios: In the control context, under-investment results from dilution of the manager’s control rights. However, if the manager owns an insignificant block of voting shares, or owns a large block with voting rights well in excess of 50%, then there is no scope for significant control dilution, and no under-investment occurs. Thus when ownership

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8In DeAngelos’ own language “Since managers and outside stockholders have voluntarily agreed to the observed allocation of voting rights, one possible interpretation of our evidence is that limited competition to manage the firm is, on net, beneficial for our sample companies.”

9For example, Burkart et al. (1998) show that the dual-class share system is optimal if the benefits of a higher likelihood of tender offers outweigh the costs of less efficient tender offers; Burkart et al. (2011) show that strong legal protection increases rivals ability to raise outside funds to finance the takeover, but absent effective competition for the target, the increased outside funding capacity does not make efficient takeovers more likely; Blair et al. (1989) show that a market for votes can increase efficiency during control contests in the presence of taxes; Burkart and Raff (2012) show that even active boards destroy acquisition opportunities for rival managers, thus forcing all firms to pay higher incentive pay to the incumbents; Neeman and Orosel (2006) show that a contest for votes in addition to a contest for shares can have efficiency advantages; Sercu and Vinaimont (2008) demonstrate with the help of simulations that one share-one vote is never optimal from an entrepreneurs point of view; and more recently, Edmans (2009) and Edmans and Manso (2011) show that shareholders that hold non-voting shares can exert influence through the threat of exit. Also, see Brav and Mathews (2011) for efficiency implication of unbundling votes using derivatives. They show that separate vote trading can improve overall efficiency in most cases.
is either very small or very large, under-investment ceases to be a problem. Managerial ownership of equity improves the alignment of management and shareholder interest, thereby reducing agency costs.

Debt also does not solve the under-investment problem, because it carries with it the risk of bankruptcy. Also, issuing more debt can require stricter covenants and this risks the incumbent manager losing control of the firm if a debt covenant is violated since the creditor could demand the manager be replaced before agreeing to a looser covenant. For example, DeAngelo and DeAngelo (1985) find evidence that dual-class firms do not resort to increasing leverage to retain control, but instead seek to keep leverage low, consistent with their desire to minimize the risk of creditors taking control of the firm. Use of nonvoting shares alleviates the under-investment problem. The issuance of nonvoting stock to fund new investment does not result in a dilution of management’s voting control and does not have any effect of debt covenants, therefore, it does not increase the likelihood of the incumbent losing control of the firm after undertaking new investments. As a consequence, the manager is more willing to invest in all available positive NPV projects. However, as noted earlier the costs associated with nonvoting equity limit its effectiveness in solving the under-investment problem.

Our analysis produces a number of testable implications for researchers in corporate finance and ownership structure. These implications are presented below:

- **Firms with greater growth opportunities are more likely to use dual-class shares.** Also, firms, whose value is determined more by the expected realization of growth options, rather than assets in place, are more likely to use dual-class shares. This implication has been, at least in part, empirically corroborated in Lehn et al. (1990), Taylor and Whittred (1998) and Dimitrov and Jain (2006).

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10 Underinvestment and its causes have been studied in a number of papers. Debt-induced under-investment has been considered by Galai and Masulis (1976), Myers (1977), and Berkovitch and Kim (1990). Risky debt in the firm’s capital structure causes the shareholders’ objectives to diverge from firm value maximization, making it optimal to forgo positive NPV projects. Myers and Majluf (1984) and Cooney and Kalay (1993) obtain conditions under which firms whose existing assets are undervalued may find it optimal to forgo positive NPV projects rather than issue underpriced equity to finance the investment.

11 Masulis et al. (2009) find that in some firms with dual-class shares, the largest shareholder does not have a majority of the voting rights. They also find evidence managers of dual-class firms with greater control wedges are likely to extract greater private benefits of control.

12 The Financial Crisis Inquiry Commission, created by U.S. Congress to examine the causes of the current financial crisis, held its first public meeting in August 2009. In his opening remarks chairman Phil Angelides discussed the vulnerability and spillover effects when debt levels reach unsustainable heights.

13 In DeAngelo and DeAngelo (1985) paper, a typical dual-class company employs almost no long-term debt (0.029 is the median debt-to-assets ratio).

14 If all the shareholders collectively take the investment decisions, then under-investment is no longer an issue; however, preventing competitors from gaining access to important information would be next to impossible. If less stringent information requirements are imposed, managers can ensure under-investment by withholding crucial information from the shareholders. Also, if it is possible to directly contract with manager, again under-investment may not be an issue. A contractual solution would also require all investment opportunities to be known to shareholders. In addition, it would require investment opportunities to be verifiable - imposing verification costs on shareholders.
• A dual-class firm is less likely to become a takeover target than a similar single-class.
A manager who issues nonvoting shares prevents the dilution of control rights and increases the role of his private benefits in a control contest. This concentration of power greatly reduces the likelihood that a potential rival will be strong enough to launch a successful takeover. This implication is consistent with the empirical results in Seligman (1986), Jarrell and Poulsen (1988), Ambrose and Megginson (1992) and Smart and Zutter (2003).

• Dual-class targets receive higher takeover premiums, which can benefit existing outside voting shareholders.
Conditional on a takeover, the premium paid for the voting shares is higher because the rival is relatively stronger and the aggregate takeover premium is distributed among a relatively smaller group of outside shareholders who owns voting shares. This implication is consistent with the empirical results in Nenova (2003) and Smart and Zutter (2003).\(^\text{15}\)

• Conditional on the same level of investment, a dual-class firm is valued less than a comparable single-class firm.
The difference between voting and nonvoting shares is that nonvoting shares cause the private benefits of managers to have a larger impact on the outcome of the control contest. If a potential rival has private benefits higher than the incumbent’s private benefits, nonvoting shares favor this rival in a control contest; conversely, if an incumbent’s private benefits are greater than the rival’s in a control contest, then nonvoting shares favor the incumbent. This implication is consistent with Claessens et al. (2002), Boone and Mulherin (2007) and Gompers et al. (2010).

• Likelihood of issuing nonvoting shares is initially increasing and then decreasing in incumbent manager’s shareholding size.
If a manager’s shareholding is sufficiently small, then there is no scope for control dilution. Thus, a manager uses voting shares to fund new investments to avoid subjecting himself (and outside shareholders) to dividend dilution and, hence, there is no under-investment problem. If a manager’s shareholding is sufficiently large, then the manager bears a large part of both the costs of under-investment and dividend dilution. Although, the issue of voting shares dilutes the effective control of the manager, given that he holds significant control rights, the cost of dilution can be outweighed by the benefits of avoiding dividend dilution, so again the manager invests in all the positive NPV projects. In both cases, the manager prefers voting shares.

• Dual-class share structures are more likely when other mechanisms that constrain a manager’s private benefits are ineffective.
\(^{15}\)Of course, the likelihood of receiving the takeover premium is lower. However, if the fall in the probability of a takeover is less than the percentage rise in the expected takeover premium, then shareholders are clearly better off in an expectation sense. See, for example, Krishnan and Masulis (2011) for a detailed discussion.
The size of private benefits depends on the manager’s ability to extract control rents and the effectiveness (or ineffectiveness) of other mechanisms in place to deter that extraction by managers (see, e.g., Dyck and Zingales (2004) and Nenova (2003)). If other deterrents are ineffective, then losing control of the firm is more costly to the manager, and as a result the manager grows more likely to forgo positive NPV projects in order to prevent control dilution. Here, outside shareholders are more likely to prefer the use of nonvoting shares to fund all possible NPV projects.\textsuperscript{16} In contrast, if the manager can preserve control through other mechanisms such as business groups, then dual-class shares become unnecessary to preserve control and continue to produce a dividend dilution effect.\textsuperscript{17}

The remainder of this article is organized as follows: In Section 2 we present the basic environment. In Section 3 we analyze the potential takeover contest and the effect of investment financing on the likelihood of potential takeover. In Section 4, we characterize the underinvestment problem. We analyze the effect of under-investment on outside shareholders as well as on the incumbent manager’s welfare. Possible extensions are discussed in Section 5. Conclusions are presented in Section 6. Some of the more cumbersome results and an extensive numerical exercise are delegated to the appendix.

\section{Model Preliminaries}

The model considers a firm that faces an investment opportunity. Our firm is a typical publicly traded firm with sizable insider holding.\textsuperscript{18} Initially, our firm has only one class of shares – the “commons.” Each common share has equal claim to cash flows as well as equal voting rights. We assume that the shareholders are able to influence, through simple majority votes, broad corporate objectives and policies such as changes in the board of directors, changes in control of the firm, and the menu of securities that the firm can issue to raise new capital. We highlight four players in our model – (i) the incumbent manager, (ii) outside shareholders, (iii) potential new investors, and (iv) the manager of a potential rival firm.

The incumbent is the one who searches for new investment opportunities, does the initial\textsuperscript{18} A study of the world’s top 27 stock markets finds that only 36\% of the largest publicly traded firms are widely held – that is, there is no single shareholder controlling more than 20\% of the total votes. Most of these widely held firms are concentrated in a few advanced markets, especially the United Kingdom, Japan, and the United States. Most large publicly traded firms (64\%) have a controlling shareholder, which may be a family (30\%), the state (19\%), or another firm (15\%). Among smaller companies the share of closely held firms is even higher. For detailed discussion see, for example, La Porta et al. (1999) and Claessens et al. (1999).

\textsuperscript{16}This implication is particularly important for countries such as India, which lacks other mechanisms (e.g., effective income and wealth disclosure for tax purposes) to monitor and constrain private benefits of control, but at the same time has enacted laws (Indian Company Act 2009) to abolish dual-class structures. Significant control-related under-investment can result.

\textsuperscript{17}Masulis et al. (2011) show that control can be maintained by pyramid structured business groups. They also show that while these organizational structures do not appear to be associated with expected rent extraction, combining this organizational structure with dual-class shares does appear to be associated with expected rent extraction.
evaluation, and decides what investment projects to undertake. We assume that the incumbent maximizes the market value of the firm as well as the private value he derives from being in control.\textsuperscript{19} In addition the incumbent is assumed to be a “block” shareholder of the firm, such that the incumbent owns $\beta$ fractions of the existing $N$ commons (or voting shares).\textsuperscript{20}

There can be two types of “closely held” ownership structures: The incumbent has a large minority block which should exceed that of any other shareholders or the incumbent has a majority of the votes. Initially, we will devote our attention to the case where the incumbent has a large minority block which should exceed that of any other shareholders; that is, $0 < \beta < 1/2$: Here, the incumbent has effective control rather than absolute control.\textsuperscript{21} The remaining $1 - \beta$ fraction of the common shares are held by outside shareholders. Each individual outside shareholder wants to maximize the value of his holdings.

The incumbent manager needs to issue equity to raise investment funds. Potential new investors are the ones who buy the securities that the firm issues, if any, to finance a new investment project. We do not restrict the existing outside shareholders from purchasing the newly issued securities, although we do assume that the incumbent manager is wealth constrained and cannot buy enough newly issued shares to keep his ownership fraction constant. Thus, if the firm invests by issuing common shares, the incumbent’s ownership fraction declines.\textsuperscript{22}

The final player is the rival manager, representing the rival firm. The rival manager, if he values our firm higher than the incumbent, offers to buy the firm. We rule out a “manager-rival negotiated” takeover: The only way to acquire the firm is through the market, in an open market purchase of at least 50% plus shares. All participants are risk-neutral and the discount rate is zero; all securities have prices equal to their expected payoff.

The temporal evolution of events is as follows: Shareholders decide on the types of securities that the firm can issue to finance the new investment opportunity. Next, the incumbent decides the level of investment, $x$, and if $x > 0$ issues securities to finance the new investment. A rival arrives, and if he can take over the firm, he bids for the firm and gains control. The actual investment is undertaken and subsequently, the firm is liquidated in the final period and the public value is paid out to the investors as a dividend. The person in control obtains the private benefits. The quality of the rival is uncertain at the beginning of the scenario, but is revealed at the time of his arrival. The figure below depicts the timeline described above.

\textsuperscript{19}These are standard assumptions in the literature. See, for example, Grossman and Hart (1988), Jensen and Meckling (1976), Myers (1977), Myers and Majluf (1984), Cooney and Kalay (1993), and Zwiebel (1996).

\textsuperscript{20}Although only about 20\% of the major exchange-listed public firms are closely held in the United States, a vast majority of U.S. corporations are closely held. According to a U.S. court, a close corporation is “typified by (1): a small number of shareholders, (2) no liquid market for corporate stock, (3) substantial majority shareholder participation in the management, directions, and operations of the corporation” (Donahue v. Rodd Electrotype Co., 367 Mass 578, 586, 328 NE2d, 505, 511 (1975)).

\textsuperscript{21}All of our results can be reproduced if we consider the case where the incumbent owns just about majority votes ($\approx 50\%$). We demonstrate this case using a numerical example in the appendix.

\textsuperscript{22}Some holders of common stock also receive preemptive rights, which enable them to retain their proportional ownership in a company should it makes a seasoned equity offering. We rule out such preemptive rights.
2.1 New Project

The project generates public value for the shareholders of the firm and a private benefit that accrues to the firm’s manager. The realized value of the project is

\[ x + a_i P(x) + \varepsilon_x. \]  

The random variable, \( \varepsilon_x \), is uniformly distributed over the interval \(( -\sigma_x, +\sigma_x )\), with a mean zero and variance \( \sigma_x^2/3 \). \( P(x) \) is a concave function, differentiable everywhere with a unique maximum at \( \bar{x} \). Thus, the maximized expected value of the new project is

\[ \bar{x} + a_i P(\bar{x}). \]  

The parameter \( a_i \) is a measure of the manager-in-control’s ability to generate cash flows from the new project. Henceforth, we call the parameter \( a_i \) “public quality” of the manager-in-control at the end of the investment process, where the manager-in-control is either the incumbent (\( I \)) or the potential rival manager (\( R \)). We assume that the public quality of the incumbent is common knowledge and \( a_I \in [0, 1] \). Initially, the potential rival manager’s public quality is unknown; thus, we assume that \( a_R \) is a random variable drawn from a uniform distribution with support 0 and 1. The lowest public quality manager is the one with \( a_i = 0 \), and the resulting NPV of the new project is 0. The highest public quality manager is the one with \( a_i = 1 \), and the resulting NPV of the new project is \( P(x) \).

Also, we assume that the manager-in-control (whether incumbent or rival) can appropriate some benefits that are not shared by outside shareholders – a private benefit of control. This private benefit is not verifiable (i.e., provable in court).\(^{23}\) The realized value of the private benefit is

\[ B_i = b_i \alpha a_i P(x), \quad i = I, R. \]  

The parameter \( b_i \) measures the manager-in-control’s ability to convert one unit of NPV into his

\(^{23}\)If it is verifiable, it will be relatively easy for outside shareholders to stop the manager from appropriating it. Thus, private benefits of control are intrinsically difficult to quantify.
private benefit. Henceforth, we will call the parameter \( b_i \) “private quality” of the manager-in-control. Like public quality, we assume that incumbent’s private quality, \( b_I \in [0, 1] \), is common knowledge and the potential rival manager’s private quality \( b_R \) is a random variable drawn from a uniform distribution with support 0 and 1. The parameter \( \alpha \) is an inverse measure of the effectiveness of “other outside private benefit monitoring mechanisms” that can help, directly or indirectly, shareholders to prevent managers from converting public value into private benefits. For example, an effective income and wealth disclosure system for income tax purposes and/or efficient legal system can act as an additional deterrence to excessive private benefits. The lower the effectiveness of such outside monitoring mechanisms; the higher is the value of \( \alpha \). An \( \alpha < 1 \) implies that the other monitoring mechanisms in place are such that even the most “villainous” managers, \( b_i = 1 \), cannot “convert” the entire NPV into private benefits.\(^{24}\)

### 2.2 Firm Value

We normalize the initial value of the firm, \( V_0 = 0 \). Hence, the present value of all the cash flows generated from the new investment is the only source of future dividends for the shareholders adjusted for the private benefit of control. There are two types of private benefits: The manager’s private benefit is direct and indirect loss to the shareholders or the manager’s private benefit is not a direct loss but only indirect loss to the shareholders. In both the cases, these private benefits are direct gains for the manager.

What is a direct costs of private benefit? One dollar worth of private benefit is one dollar less for the outside shareholders (value effect). What are the indirect costs of private benefits? Sometimes private benefit does not directly reduce value of the firm, but allows the manager to use these proceeds from private benefits to stall potential value-enhancing takeovers (entrenchment effect). First we will focus on the case which is a mixture of these two effects: Private benefits reduces firm value as well as helps to entrench the manager. In Section 7 of the paper, we develop an extensive numerical example that deals only the “entrenchment effect” of private benefits on investment decision.

If the firm invests in this new project and the manager-in-control is of \((a_i, b_i)\) type, then the expected firm value, denoted by \( FV_i \), is

\[
FV_i = x + a_i P(x) - b_i \alpha a_i P(x) = x + a_i (1 - b_i \alpha) P(x) = x + \hat{a}_i P(x),
\]

(4)

where \( i = I, R \) and \( \hat{a}_i = a_i (1 - b_i \alpha) \).\(^{25}\) We will start by stating one basic result in the proposition below.

**Proposition 1.** From outside shareholder’s point of view, the ideal manager has public quality \( a_i = 1 \), and private quality \( b_i = 0 \).

\(^{24}\)See, for example, Dyck and Zingales (2004) and Nenova (2003) for detailed discussions on private benefits.

\(^{25}\)We assume that \( E(a_i b_i) = E(a_i) E(b_i) \), implying that \( a_i \) and \( b_i \) are independent.
Proof. Follows directly from equation (4).

Outside shareholders want to maximize the firm value, \( FV_i \). Given that private benefit for the manager-in-control is a one-for-one loss to the shareholders’ value, outside shareholders want a manager who is best in terms of public quality and has the least ability to convert shareholders’ value into private benefits; that is, a manager with \( a_i = 1 \) and \( b_i = 0 \). If \( a_i = 1 \) and \( b_i = 0 \), the “best quality” manager is in control of the firm, then the expected firm value, \( x + P(x) \) for any level of investment \( x \), is maximized.

If \( a_i = 0 \) and \( b_i = 1 \), the “worst quality” manager is in control of the firm. Here the expected firm value \( x \), is equal to the invested funds. If \( a_i = 1 \) and \( b_i = 1 \), the manager-in-control is the best manager in terms of public quality but at the same time extracts large amounts of private benefits, so that the resultant firm value is \( x < x + (1 - \alpha)P(x) < x + P(x) \). There must be a potential rival with lower public quality and private quality than the incumbent, \( 0 < a_R < 1 \) and \( b_R < 1 \), such that \( a_R (1 - b_R \alpha) > (1 - \alpha) \). This condition implies that if this potential rival takes over control of the firm, he can generate higher value for the outside shareholders than the incumbent. For example, if we follow Dyck and Zingales (2004)’s average estimate of shareholders’ wealth appropriation by the controlling shareholder and set \( \alpha = 0.14 \), we get that a potential rival manager with \( a_R \geq 0.925 \) and \( b_R = 0.5 \) can generate higher cash flows than the incumbent with \( a_I = 1 \) and \( b_I = 1 \).

3 Potential Control Contest

The potential control contest is a critical stage of the process and we explain it further. To gain control of the firm, the rival has to offer shareholders a greater amount for their shares than the incumbent. If the rival cannot offer a greater amount, he does not bid and the incumbent retains control. If the rival can offer a greater amount, he pays shareholders an amount equal to (or slightly higher than) what the incumbent can offer and the firm comes under the rival’s control. Initially, we assume that the incumbent does not tender in a control contest. This is because firm insiders’ stock sells are subject to restrictions by supervisory authorities and these restrictions may severely affect the manager’s ability to tender in a control contest. For example, the incumbent manager may hold “restricted voting” shares, which will prevent the manager from tendering in a control contest.

Shares derive value from two sources: The first source is the public value, which shareholders receive in the form of dividends when the firm is liquidated. This is referred to also as the “value of the dividend.” The second source is the private value or takeover premium that is extracted

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26 If we take the upper estimates of Dyck and Zingales (2004) and set \( \alpha = 0.65 \), then an average quality potential rival \((a_R = 0.52 \text{ and } b_R = 0.5)\) can generate more cash flows than the incumbent with \( a_I = 1 \) and \( b_I = 1 \).

27 In Subsection 5.1 we relax this assumption and allow the manager to tender if a rival makes an offer. We show that qualitatively similar results are obtained if the incumbent is allowed to tender in the control contest.
in a control contest and paid from the private benefits of the manager-in-control. This value accrues only to shareholders who can vote in the takeover contest and is realized only when there is a change in control. We refer to it as the “value of vote” hereafter.\footnote{Our analysis is restricted to two types of securities – nonvoting shares (the holders of these securities are only entitled to a share of the public value of the firm; they do not get to vote on a takeover) and “commons” or voting shares (these securities entitle their holders to a share in the public value of the firm and to vote on changes in control). The effect of multiple classes of securities and the problem of optimal security design (the “best” combination of dividend and vote) are not formally addressed here. A discussion later in the paper addresses these issues.}

At this stage we introduce additional notation to make the problem easier to understand. A superscript \( j \in \{0, 1\} \) on variables indicates the value of the variable if the firm issues new shares with \( j \) votes per-share; e.g., \( j = 0 \) corresponds to nonvoting common shares, while \( j = 1 \) corresponds to voting common shares. Let

- \( n^j \) = number of new shares issued if \( j \)-vote shares are issued to finance the investment;
- \( \phi^j \) = probability of no takeover, if \( j \)-vote shares are issued to finance the investment;
- \( V^j_D \) = public value per-share if \( j \)-vote shares are issued to finance the investment;
- \( V^j_{\text{vote}} \) = value of a pure vote claim if \( j \)-vote shares are issued to finance the investment.

The value per-share of the \( k \)-vote shares when \( j \)-vote shares are issued to finance the investment, \( V^j_k \), is important. If the new investment is financed using voting shares, only one type of share is outstanding and their value is denoted by \( V^1_1 \). If nonvoting shares are issued to finance investment, then two different types of share are outstanding and their values are given by \( V^0_1 \), for the old voting shares, and \( V^0_0 \), for the newly issued nonvoting shares.\footnote{\( V^1_0 \) is not relevant because all existing shares are voting shares by assumption, and if the new shares issued are also voting shares, nonvoting shares do not exist.} The value of the voting shares is equal to the value of the dividend received plus the value of the vote, while the value of the nonvoting shares is equal to just the value of the dividend received. Thus, the value of the voting shares, if new shares issued to finance the investment are voting shares, is

\[
V^1_1 = V^1_D + V^1_{\text{vote}},
\]

and the value of the voting shares, if new shares issued are nonvoting shares, is

\[
V^0_1 = V^0_D + V^0_{\text{vote}}.
\]

If nonvoting shares are issued to finance the investment, then the value of each nonvoting share is equal to just the value of the dividend received,

\[
V^0_0 = V^0_D.
\]
to nonvoting. That is, \( V_D^1 \neq V_D^0 \). This is because the number of new shares that the firm has to issue to finance \( x \) dollars of investment, \( n^j = \frac{x}{V_i^j} \), depends on the type of security issued.

**Proposition 2.** The number of new nonvoting shares, \( n^0 \), issued to raise \( x \) dollars is always at least weakly greater than the number of voting shares, \( n^1 \), needed to be issued to raise the same dollar amount.

**Proof.** Follows directly from equations (5), (6) and (7).

By design, voting shares and nonvoting shares have equal dividends. Since the value of vote is nonnegative (vote premium is exactly like option premium), the value of one voting share, which is equal to the expected value of dividend plus the value of vote, has to be at least weakly greater than value of one nonvoting share, which is just the expected value of dividend. Therefore, the number of voting shares issued to raise \( x \) dollars is at least weakly less than the number of nonvoting shares issued to raise the same \( x \) dollars.

### 3.1 The Decision Problems

The two decision problems can now be formally set up. Both the manager and the outside shareholders are assumed to be interested in maximizing their expected wealth. For the manager the decision variable is the level of investment, \( x \). Given that the manager does not tender his shares to the rival, this level is equivalent to

\[
\max_x \left[ \beta N V_D^j (x) + \phi^j b_I a_I P(x) \right].
\]

(8)

The objective function above has two parts: The first part is related to the public value of the firm and reflects the fact that the manager is similar to any other shareholder. The second part is related to the private benefit of incumbency, and is realized only if the manager retains control of the firm. The solution to the manager’s problem gives the manager’s optimal response to restrictions on the type of security that the firm can issue.

Let \( \hat{x}^j \) be the solution to the manager’s optimization problem given that he issues \( j \)-vote shares to finance the investment. Outside shareholders maximize the value of their shares, picking the type of security that the manager can issue and taking the manager’s optimal response as given. Thus, the decision problem of the outside shareholders is

\[
\max_{j=0,1} V_i^j (\hat{x}^j).
\]

(9)

To solve the two optimization problems, given by equations (8) and (9), we need (i) the probability that there is no takeover, (ii) the value of the dividend, and (iii) the value of the vote for

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30 See, for example, Smith and Amoako-Adu (1995) for discussion on relative prices of voting and nonvoting shares.
the case where the firm issues voting shares and for the case where the firm issues nonvoting shares.

3.2 Potential Control Payoffs

A change in control occurs when the rival can offer a higher per-share value to the outside shareholders than the incumbent. The probability of a takeover can be obtained by considering separately the cases of voting shares and nonvoting shares. The incumbent can retain control if he can offer more for the shares than the rival. If voting shares are used to finance the investment, this condition is equivalent to

\[
\frac{x + a_I P(x)}{N + n^1} - \frac{b_I \alpha a_I P(x)}{N + n^1} + \frac{b_I \alpha a_I P(x)}{(1 - \beta) N + n^1} \geq \frac{x + a_R P(x)}{N + n^1} - \frac{b_R \alpha a_R P(x)}{N + n^1} + \frac{b_R \alpha a_R P(x)}{(1 - \beta) N + n^1}.
\]

The first two terms on the LHS of equation (10) describe the per-share public value that is generated with the incumbent in control. The third term on the LHS is related to the incumbent’s private value. Its denominator is smaller than the first two terms’ because the private benefit is distributed only to the outside shareholders. The RHS terms are related to the public and private benefit per-share generated under the rival. Simplifying equation (10) we get

\[
a_I \left(1 + \alpha \kappa^1 b_I \right) \geq a_R \left(1 + \alpha \kappa^1 b_R \right),
\]

where \(\kappa^1 = \frac{N \beta}{(1 - \beta) N + n^1}\).

If nonvoting shares are issued to finance the investment, then the incumbent retains control if

\[
\frac{x + a_I P(x)}{N + n^0} - \frac{b_I \alpha a_I P(x)}{N + n^0} + \frac{b_I \alpha a_I P(x)}{(1 - \beta) N} \geq \frac{x + a_R P(x)}{N + n^0} - \frac{b_R \alpha a_R P(x)}{N + n^0} + \frac{b_R \alpha a_R P(x)}{(1 - \beta) N}.
\]

The private value is distributed only to those outside shareholders who own voting shares. The holders of the nonvoting shares do not get a share in the private value because they cannot affect the outcome of the control contest. Simplifying equation (12) we get

\[
a_I \left(1 + \alpha \kappa^0 b_I \right) \geq a_R \left(1 + \alpha \kappa^0 b_R \right),
\]

where \(\kappa^0 = \frac{N \beta + n^0}{(1 - \beta) N}\). From equations (11) and (13), it is clear that it is not possible for any rival to take over the firm from an incumbent using the corporate control market if the incumbent has very high public as well as private qualities. This is formally stated in the proposition below.

**Proposition 3.** Market based change in corporate control is not possible if the incumbent manager is best in terms of public quality, \(a_I = 1\), and worst in terms of private quality, \(b_I = 1\).

**Proof.** Follows directly from equations (11) or (13).
This is an interesting result: An incumbent, who is the best-possible manager in terms of public quality and the worst-possible manager in terms of private quality can offer the maximum acquisition price in any potential control contest. Hence, a market-based takeover of the firm is not possible, even though the incumbent generates much lower shareholder value than a host of potential rivals. In a sense, market for corporate control fails to work! And this result is independent of the type of security incumbent uses to fund the new project. Thus, the firm value, $\bar{x} + (1 - \alpha)P(\bar{x})$, is always less than the maximum possible value, $\bar{x} + P(\bar{x})$. After simplifications, equations (11) and/or (13) can be expressed as

$$b_R \leq b_R^j = \frac{1}{\alpha \kappa^j} \left( \frac{a_I}{a_R} - 1 \right) + b_I \frac{a_I}{a_R}, \quad j = 0, 1,$$

where $b_R^j$ is the lowest value of $b_R$ such that takeover is not possible. Given that $b_R^j \in [0, 1]$, we simplify and rearrange equation (14) to define $\bar{a}_R^j$ and $\bar{a}_R^j$:

$$\bar{a}_R^j = a_I(1 + \alpha \kappa^j b_I) \quad \text{and} \quad \bar{a}_R^j = \frac{a_I(1 + \alpha \kappa^j b_I)}{1 + \alpha \kappa^j} = \frac{\bar{a}_R^j}{1 + \alpha \kappa^j}. \quad (15)$$

The likelihood of takeover and the role of private benefit in a takeover contest depends on $\bar{a}_R^j$, $\bar{a}_R^j$ and $b_R^j$. Proposition below formally states these observations.

**Proposition 4.** (i) Rivals with public quality $a_R$ higher than $\bar{a}_R^j$ can gain control of the firm irrespective of their private quality (i.e., even if $b_R = 0$); whereas (ii) rivals with public quality lower than $\bar{a}_R^j$ cannot gain control of the firm, even if they have the highest possible private quality ($b_R = 1$).

**Proof.** Directly follows from equations (14) and (15). □

If the potential rival’s public quality is significantly higher (lower) than the public quality of the incumbent, then the control contest will be decided based only on the public quality of the contestants, and their private qualities will not play any role in the control contest.\(^{31}\) If the rival manager’s public quality is high ($a_R > \bar{a}_R^j$), then the rival manager gains control of the firm; whereas, if the rival manager’s public quality is low ($a_R < \bar{a}_R^j$), then the incumbent retains control of the firm.\(^{32}\)

Private qualities of the contestants play a role in the control contest only when potential rivals are drawn from the set of public quality, $a_R \in [\bar{a}_R^j, \bar{a}_R^j]$. Rival managers with public quality $a_R \in [\bar{a}_R^j, \bar{a}_R^j]$ and private quality $b_R \in [b_R^j, 1]$ can takeover control of the firm. These control regions, based on potential rival’s public quality and private quality are depicted in Figure 1 below.

\(^{31}\)Possibly there will be no control contest. These situations are potential candidates for “manager-rival negotiated” takeovers.

\(^{32}\)There can be many $(a_I, b_I)$ combinations such that $\bar{a}_R^j > 1$. This simply implies that no potential rival exists who can take over the firm based only on his public quality.
Figure 1: This figure depicts four distinct parameter ranges based on a potential rival’s public quality and private quality critical for the control contest. If the potential rival’s public quality is drawn from rectangular region I, the incumbent loses control of the firm irrespective of the rival manager’s private quality. Similarly, if a potential rival’s public quality is drawn from rectangular region III, the incumbent retains control of the firm irrespective of the rival manager’s private quality. Hence, private benefit does not play any role in the takeover contest if the potential rival’s public quality belongs to either \([a^j_R, 1]\) or \([0, a^j_R]\). If the potential rival is drawn from the range \([a^j_R, \bar{a}^j_R]\), then only private benefit of the rival manager plays a role in the takeover contest. If the potential rival’s private quality, \(b_R > b^j_R\) (dotted line), then the incumbent loses control of the firm. Otherwise, the incumbent retains control.

3.3 Effect of Investment on Control Contest

Next, consider the effects of increasing investment, \(x\), on \(\bar{a}^j_R\), \(\bar{a}^j_R\), and \(b^j_R\). These bounds determine the outcome of the control contest: Likelihood that incumbent retains control of the firm after investment depends on the level of investment and the incumbent’s ability to choose the size of \(x\). Since \(\kappa^j\) appears in all three expressions, let us first see the effect of increasing investment on \(\kappa^j\). The number of new shares needed to finance the investment increases as the size of the investment increases. We show in appendix \(B\) that \(\frac{\partial n^j}{\partial x} > 0 \forall j\), implying that \(\frac{\partial \kappa^0}{\partial x} > 0\) and \(\frac{\partial \kappa^1}{\partial x} < 0\). Thus,

\[
\frac{\partial}{\partial x} \bar{a}^j_R = \alpha a_I b_I \frac{\partial \kappa^j}{\partial x} \begin{cases} > 0 & \text{if } j = 0 \\ < 0 & \text{if } j = 1, \end{cases} \tag{16}
\]

and

\[
\frac{\partial}{\partial x} a^j_R = \frac{\partial}{\partial \kappa^j} a^j_R \times \frac{\partial \kappa^j}{\partial x} = -\frac{a_I (1 - b_I) \alpha}{(1 + \alpha \kappa^j)^2} \times \frac{\partial \kappa^j}{\partial x} \begin{cases} < 0 & \text{if } j = 0 \\ > 0 & \text{if } j = 1 \end{cases} \tag{17}
\]

Effects of increasing \(x\) on \(\bar{a}^j_R\) and \(a^j_R\) are important for the potential control contest. Hence, we formally state these results in the propositions 5 and 6 below.

**Proposition 5.** (i) If the incumbent funds these new projects by issuing voting shares, then
the set of rivals who can take over the firm using only their public quality, \([\bar{a}_R^i, 1]\), increases; whereas (ii) if the incumbent funds these new projects by issuing nonvoting shares, then the set of rivals who can take over the firm using only their public quality, \([a_0^i, 1]\), decreases.

**Proof.** Follows directly from equation (16).

If the incumbent uses voting shares to fund the new project, then the larger the investment, \(x\), the larger the chance that a rival can gain control of the firm irrespective of his private quality (i.e., even if \(b_R = 0\)). An incumbent with relatively high private quality will be worried about this result: As he issues more and more voting shares to raise funds for investment, his private benefits become less and less useful in the control contest! This is because voting shares shift the relative weights from vote premium to dividends. The vote premium, which is paid using his private benefits, is divided among a larger number of outside shareholders, \((1 - \beta)N + n^1\), and as a result the per-share vote premium falls. In contrast, if the incumbent issues more and more nonvoting shares to raise funds for investment, the importance of private benefits remains unchanged; because, these newly issued shares are nonvoting and, hence, have no claim on the vote premium. The Incumbent can use his private benefits to buy just the existing outside votes, \((1 - \beta)N\).

**Proposition 6.** (i) If the incumbent funds these new projects by issuing voting shares, then the region over which he retains control of the firm irrespective of the rival’s private quality, \([0, a_1^i]\), increases; whereas (ii) if the incumbent funds the new project issuing nonvoting shares, then the region over which he retains control of the firm irrespective of rival’s private quality, \([0, a_0^i]\), decreases.

**Proof.** Follows directly from equation (17).

If the incumbent uses voting shares to fund the new projects, then the role of his private benefit in the takeover contest diminishes (region II gets smaller). This is because issuing voting shares shifts the weight from vote premium to dividends, and thereby reduces the role of private benefits. On the other hand, if the incumbent uses nonvoting shares to fund the new projects, then the role of his private benefits in the control contest increases (region II expands); issuing nonvoting shares shifts the weight from dividends to vote premium, and thereby increases the importance of private benefits. Panel A and panel B of Figure 2 depict the results stated in Propositions 5 and 6.

Suppose \(a_R \in [a_R^i, a_R^j]\); hence, the feasibility of the takeover depends also on the rival’s private quality, \(b_R\). Thus, we need to check what happens to the minimum private quality, \(b_R^j\), required for successful takeover as \(x\) increases. Differentiating LHS expression of equation (14) with respect to \(x\), we get

\[
\frac{\partial}{\partial x} b_R^j = \frac{1}{\alpha (\kappa^j)^2} \left( \frac{a_I}{a_R} - 1 \right) \frac{\partial k^j}{\partial x}.
\]
Hence, the sign of $b^j_R$ is not unambiguous – it depends on the relative values of $a_I$ and $a_R \in [a^j_R, \tilde{a}^j_R]$. If public quality of the incumbent is smaller than the public quality of the potential rival, $a_I < a_R \in [a^j_R, \tilde{a}^j_R]$, then

$$
\frac{\partial}{\partial x} b^j_R = \frac{1}{\alpha (K^j)^2} \left( \frac{a_I}{a_R} - 1 \right) \frac{\partial k^j}{\partial x} \begin{cases} > 0 & \text{if } j = 1 \\ < 0 & \text{if } j = 0. \end{cases}
$$

(19)

Similarly, the effect of increasing investment, $x$, on the likelihood of incumbent’s retaining control is also not obvious. Region I, where the incumbent loses control expands, but at the same time region III, where the incumbent retains control expands too. The net effect depends on incumbent’s public and private quality relative to the average public and private qualities of the potential rival.

### 3.4 Probability of Control

The probability of the incumbent’s retaining control of the firm after issuing either voting shares ($j = 1$) or nonvoting shares ($j = 0$) is

$$
\phi^j(x) = \int_{a^j_R}^{\tilde{a}^j_R} da_R + \int_{a^j_R}^{\tilde{a}^j_R} \int_{0}^{b^j_R} db_R da_R \quad \text{where } j = 0, 1.
$$

(20)

The first term is the region where the potential rival’s public quality is very low. In this region the rival has no hope of gaining control regardless of his private quality. The second term is the
region where the rival’s public quality is such that the incumbent retains control if the rival’s private quality is lower than \( b_R \); otherwise, the rival gains control. There is one more range \([\bar{a}_R, 1]\), but the incumbent has no hope of retaining control over this range, irrespective of the private quality of the rival. Integrating the expression in equation (20) and further simplifying, we get the probability of the incumbent’s retaining control,

\[
\phi^j(x) = \left( a_I(1 + b_I k^j \alpha) \frac{\log(1 + k^j \alpha)}{k^j \alpha} \right).
\]

(21)

If we differentiate equation (21) with respect to \( x \) using the chain rule, we get

\[
\frac{\partial \phi^j(x)}{\partial x} = \frac{\partial \phi^j(x)}{\partial k^j} \times \frac{\partial k^j}{\partial x} = a_I \left( \frac{\frac{\log(1 + k^j \alpha)}{\alpha} - \frac{\log(1 + k^j \alpha)}{\alpha}}{(k^j)^2} \right) \times \frac{\partial k^j}{\partial x}.
\]

(22)

For all \( b_I \geq 1/2 \), \( \frac{\partial \phi^j(x)}{\partial x} \) is nonnegative and for \( b_I < 1/2 \), \( \frac{\partial \phi^j(x)}{\partial x} \) is strictly negative. Hence, the sign of \( \frac{\partial \phi^j(x)}{\partial x} \) depends on the sign of \( \frac{\partial k^j}{\partial x} \) and value of \( b_I \). Thus,

\[
\frac{\partial \phi^1(x)}{\partial x} = \frac{\partial \phi^1(x)}{\partial k^1} \times \frac{\partial k^1}{\partial x} \begin{cases} < 0 & \text{if } b_I \geq 1/2 \\ > 0 & \text{if } b_I < 1/2. \end{cases}
\]

(23)

Propositions 7 and 8 describe the relationships among the likelihood of retaining control of the firm, private quality of the incumbent, and the level of investment.

**Proposition 7.** (i) If the incumbent manager has relatively high ability to extract private benefit, \( b_I \geq 1/2 \), and issues voting shares to fund these new projects, then the likelihood of the incumbent’s retaining control of the firm decreases as the level of investment increases.

(ii) If the incumbent has relatively low private quality, \( b_I < 1/2 \), and issues voting shares to fund these new projects, then the likelihood of the incumbent’s retaining control of the firm increases as the level of investment increases.

**Proof.** Follows directly from equation (23).

The incumbent’s private quality relative to the average private quality of potential rival managers, \( E(b_R) = 1/2 \), plays an important role in the control contest. If the incumbent is better in terms of private quality than the rival’s pool, using voting shares to fund the new investment projects reduce his chance of retaining control of the firm after the investments. This is because voting shares make dividends as opposed to takeover premium more important in a control contest. Few voting shares are needed to be issued; hence, dividend dilution is low but the private benefit of the potential rival gets distributed among the old as well as the new voting shares. Similarly,

\[
\frac{\partial \phi^0(x)}{\partial x} = \frac{\partial \phi^0(x)}{\partial k^0} \times \frac{\partial k^0}{\partial x} \begin{cases} > 0 & \text{if } b_I \geq 1/2 \\ < 0 & \text{if } b_I < 1/2. \end{cases}
\]

(24)
Proposition 8. (i) If the incumbent has relatively high ability to extract private benefit, \( b_I \geq 1/2 \), and uses nonvoting shares to fund these new projects, then the likelihood of the incumbent’s retaining control of the firm increases as the level of investment increases.

(ii) If the incumbent has relatively low private quality, \( b_I < 1/2 \), and uses nonvoting shares to fund these new projects, then the likelihood of the incumbent’s retaining control of the firm decreases as the level of investment increases.

Proof. Follows directly from equation (24).

Nonvoting shares make takeover premium more important than dividends in a control contest. Thus, an incumbent with relatively high private quality can use his private benefits to boost his chance of retaining control of the firm in case of a control contest. In Figure 3 we depict the likelihood that the incumbent retains control of the firm post-investment under voting share financing and nonvoting share financing.

![Graph](image.png)

**Figure 3**: This figure depicts the likelihood that the incumbent retains control of the firm, \( \phi^j \), as a function of the size of the new equity issue, \( n^j \). The fixed parameters used to generate the figure are as follows: \( a_I = 0.5 \), \( \alpha = 0.85 \), \( N = 100 \), and \( \beta = 0.30 \). In panel A, we plot the incumbent’s likelihood of retaining control when the new investment is financed using voting shares. In panel B, we plot the incumbent’s likelihood of retaining control when the new investment is financed using nonvoting shares. The solid black lines, in both panels A and B, correspond to the case where the incumbent’s private quality is high (\( b_I = 0.8 > 0.5 \)), and the dashed lines, in both panels A and B, correspond to incumbent’s private quality is relatively low (\( b_I = 0.2 < 0.5 \)).

Everything else remaining the same, the incumbent’s likelihood of retaining control is higher, the higher his private quality. In both panel A and B of Figure 3, the **solid line** is above the **dashed line**. If the manager chooses not to invest, implying \( n^j = 0 \), then for an incumbent with \( a_I = 0.5 \) and \( b_I = 0.8 \), the likelihood of retaining control of the firm is 0.566; in contrast, for an incumbent with \( a_I = 0.5 \) and \( b_I = 0.2 \), the likelihood of retaining control of the firm is 0.441. Obviously, an incumbent’s likelihood of retaining control of the firm depends on his public quality: – For example, if \( a_I = 0.8 \) and \( b_I = 0.8 \), then the likelihood that the incumbent will retain control is 0.881.

If the incumbent with relatively high private quality (\( b_I > 1/2 \)) uses voting shares to fund the new investment, then the likelihood that the incumbent retains control of the firm decreases
as \( x \) increases.\(^{33}\) This relationship between \( x \) and \( \phi^1 \) holds for all \( a_I \). In contrast, if the incumbent with relatively low private quality \((b_I < \frac{1}{2})\) chooses to invest using voting shares; then the likelihood that the incumbent retains control of the firm increases as \( x \) increases and the relations holds for all \( a_I \).

The relationship between investment and the likelihood of retaining control is just opposite if the incumbent uses nonvoting shares to fund the new investment. If his private quality is relatively high (low), then the likelihood that the incumbent retains control increases (decreases) as \( x \) increases (decreases), and this relationship between \( x \) and \( \phi^0 \) holds for all \( a_I \).

### 3.5 Value of One Dividend Claim

The value of a pure dividend claim is equal to the expected dividend that the holder of the claim gets. Obviously, this value depends on the manager-in-control’s public quality and the type of security issued to finance the new investment. Thus, the value of the per-share pure dividend claim is

\[
V_D^j = \phi^j \cdot \frac{FV_I}{N + n^j} + \int_{\frac{a^j_R}{2}}^{\frac{b^j_R}{2}} \int_{\frac{b^j_R}{2}}^{1} \frac{FV_R}{N + n^j} \, db_R \, da_R + \int_{\frac{b^j_R}{2}}^{1} \int_{0}^{1} \frac{FV_R}{N + n^j} \, db_R \, da_R \quad \text{where } j = 0, 1. \quad (25)
\]

The first term is the probability that the incumbent retains control times the per-share public value of the firm under the incumbent. The second and third terms give the expected dividend under the rival. The second term is generated by a rival of intermediate public quality and relatively high \( (> \frac{b^j_R}{2}) \) private quality. The third term is generated by rivals of very high public quality, who can take over the firm regardless of their private quality. If \( \phi^j \) decreases, then \( 1 - \phi^j \) increases and vice versa.

Given our assumption that all available projects are positive NPV projects, the level of dividend increases with the level of investment, \( x \). If \( x \) increases, either it is more likely for the incumbent to retain control or it is more likely for the rival to wrest control from the incumbent. If nonvoting shares are used to fund the new investment, then the average dividend per-share can increase less than if voting shares are used. This is because the number of new nonvoting shares required to raise \( x \) is greater than the number of voting shares; hence, each dividend claim gets diluted.

### 3.6 Value of One Vote Claim

Voting rights matter because they allow stockholders to have a say in who runs the company and how it is run.\(^{34}\) It is true that most stockholders don’t use these rights and prefer to vote

\(^{33}\)As \( x \) increases so does \( n^j \) (not a linear relationship but monotonic); hence, we use \( n^j \) in the horizontal axis of Figure 3.

\(^{34}\)Papers such as Zingales (1995b) and Nenova (2003) have quantified the value of the vote based on the price difference of shares with disparate voting rights of firms that have both classes traded. They find that the value
with their feet, but voting power does come into use, especially at badly managed companies, when a challenge is mounted against the incumbent either from within (activist stockholders) or from outside (hostile acquisitions).\(^{35}\)

The value of a pure vote claim is related to the extraction of private benefits from the rival in the form of the takeover premium. To obtain an expression for the value of the vote, we classify rival managers into one of three types: The first type represents rivals who cannot gain control of the firm because they have very low public quality \((a^R < a^j_R)\). If this type of rival is drawn, no private benefit is extracted and the value of the vote is zero. Next consider the rivals who can gain control of the firm without having to pay out any of their private benefit – those with very high public quality \((a^j_R > \bar{a}^j_R)\). Again, it is not necessary for the rival manager to give up any of his private benefits. Hence, the private benefit is extracted only when a rival manager is of an intermediate type \((a^1_R < a^j_R < \bar{a}^j_R)\). The payoff to the vote claim when the firm issues voting shares can be written as

\[
\begin{cases}
    \frac{FV_I}{N + n^1} + \frac{B_I}{N(1 - \beta) + n^1} - \frac{FV_R}{N + n^1} & \text{if } a^1_R \leq a_R \leq \bar{a}^1_R \\
    0 & \text{otherwise.}
\end{cases}
\tag{26}
\]

Similarly, the payoff to the vote claim when the firm issues nonvoting shares is

\[
\begin{cases}
    \frac{FV_I}{N + n^0} + \frac{B_I}{N(1 - \beta) + n^0} - \frac{FV_R}{N + n^0} & \text{if } a^0_R \leq a_R \leq \pi^0_R \\
    0 & \text{otherwise,}
\end{cases}
\tag{27}
\]

where \(B_I = b_I \alpha a_I P(x)\) is the total private benefit of the incumbent given a level of investment, \(x\), and the incumbent stays in control. Similarly, \(B_R = b_R \alpha a_I P(x)\) is the total private benefit of the potential rival given a level of investment, when the rival wins the control contest. The value of the vote is simply the expectation of these values,

\[
V_{\text{vote}}^1 = \left( \frac{FV_I}{N + n^1} + \frac{B_I}{N(1 - \beta) + n^1} \right) \int_{a^1_R}^{\bar{a}^1_R} \int_{b^1_R}^{1} db_R da_R - \int_{a^1_R}^{\bar{a}^1_R} \int_{b^1_R}^{1} \frac{FV_R}{N + n^1} db_R da_R \tag{28}
\]

and

\[
V_{\text{vote}}^0 = \left( \frac{FV_I}{N + n^0} + \frac{B_I}{N(1 - \beta)} \right) \int_{a^0_R}^{\bar{a}^0_R} \int_{b^0_R}^{1} db_R da_R - \int_{a^0_R}^{\bar{a}^0_R} \int_{b^0_R}^{1} \frac{FV_R}{N + n^0} db_R da_R. \tag{29}
\]

of the vote is positive and varies across countries. See also Smart et al. (2008) for implications of “vote” on IPO valuation.

\(^{35}\)Institutional investors’ benign neglect of different voting share classes at Google is rationalized by the fact that they think the company is well managed and that control is therefore worth little or nothing. There is a kernel of truth to this statement: The expected value of control (and voting rights) is greater in badly managed companies than in well managed ones. However, if you are an investor for the long term, you have to worry about whether managers who are perceived as good managers today could be perceived otherwise in a few years.
Using equations (5) and (6), we can derive the value of voting shares when the new shares issued are voting, as well as when the new shares issued are nonvoting shares. Next, we depict the type of manager who will underinvest if he is forced to finance investment using voting shares.

4 Entrenchment and Investment

The manager chooses the investment level to maximize his expected wealth. There are three terms in the manager’s objective function that depend on the level of investment chosen: The value of the dividend, the probability of retaining control, and the private benefits of control. The value of the dividend increases with investment as we consider only positive NPV opportunities. By design, the private benefits of control also increase with investment. We also see, from equations (23) and (24), that the likelihood that the incumbent retains control of the firm depends primarily on two things: Level of investment \(x\) and private quality of the incumbent \(b_I\).

4.1 Investments Financed Using Voting Shares and Nonvoting Shares

First, let us consider the case in which the manager does not own any equity in the firm. Suppose investments are financed by issuing voting shares. Because the manager owns no equity in the firm, there is no possibility of dilution in his ownership and the probability of the manager’s retaining control is unaffected by the level of investment chosen. Thus, the manager’s objective function is strictly increasing in \(x\) and as a result the incumbent invests in all available positive NPV projects. The proposition below formalizes this result.

**Proposition 9.** When investments are financed by issuing voting shares and the incumbent does not own any equity in the firm (i.e., \(\beta = 0\)), then the incumbent invests in all available positive NPV projects.

**Proof.** See the proof stated in Section 8.2.

This is a counter-intuitive result: When the incumbent owns part of the firm’s equity, he bears part of the cost of under-investment. The larger the incumbent’s ownership, \(\beta\), the larger his share of the cost of under-investment. Thus, at a glance, it seems that if \(\beta = 0\), the incumbent cares only about the probability of retaining control and his private benefits; consequently, it seems plausible that the level of under-investment will be more severe! But the above result shows that if the incumbent’s ownership, \(\beta\), equals 0, then the incumbent always invests in all available positive NPV projects. From equation (10), we see that if \(\beta = 0\), then it does not matter whether the incumbent pays more dividends or pays some as vote premium, because the number of outside votes, \(N + n^1\), is equal to number of dividend claims, \(N + n^1\). Hence,
the control contest depends only on how much cash flow the incumbent generates vis-à-vis the potential rival. Thus, he invests \( \bar{x} \) to maximize the cash flows and as a result maximizes the likelihood that he will retain control.

Next, we consider the firm’s financing decision and its effect on the incumbent’s likelihood of retaining control of the firm after the new issue is completed. From equations (23) and (24) we know that the private quality of the incumbent, \( b_I \), determines whether the probability of retaining control, \( \phi_j \), increases or decreases with the level of investment. Given any level of public quality of the incumbent, \( a_I \), we can isolate two types of incumbent: The incumbent with relatively high private quality, \( 1/2 \leq b_I \leq 1 \); and the incumbent with relatively low private quality, \( 0 \leq b_I < 1/2 \).

For the incumbent with relatively high private quality, issuing nonvoting shares to fund the new investments causes the incumbent’s likelihood of retaining control of the firm, \( \phi^0 \), to increase with the level of investment, \( x \); whereas, for the incumbent with relatively low private quality, issuing nonvoting equity to fund the new project causes his likelihood of retaining control of the firm, \( \phi^0 \), to decrease with the level of investment, \( x \). In contrast, if the incumbent has relatively low private quality, and uses voting shares to fund the new investments, then the likelihood that he retains control increases with the level of investment, \( x \); whereas if the incumbent has relatively high private quality, and uses voting shares to fund the new investments, then the likelihood that the incumbent retains control increases with the level of investment, \( x \). The proposition below formalizes this result in the context of achieving full employment.

**Proposition 10.** If the incumbent manager has relatively low private quality, \( 0 \leq b_I < 1/2 \), and uses voting shares to fund these new projects, then the incumbent invests \( \bar{x} \). Similarly, if the incumbent manager has relatively high private quality, \( 1/2 \leq b_I \leq 1 \), and uses nonvoting shares to fund these new projects, then also the incumbent invests \( \bar{x} \).

**Proof.** See the proof stated in Section 8.3.

The intuition is straightforward: If the incumbent has relatively low private quality, \( b_I < 1/2 \), then the incumbent on an average expects a rival who is superior to him in terms of private quality. This is because the unconditional expectation of the potential rival’s private quality is \( 1/2 \), and it is greater than his own private quality, \( b_I \). Hence, by raising the level of investment by using voting shares and thereby increasing the number of shares entitled to takeover premium, the incumbent makes the control contest relatively more dependent on dividend vis-à-vis private benefits. This makes it difficult for the rival to take over the firm using his private quality. Thus, the incumbent takes up all available positive NPV projects.

Similarly, if the incumbent has relatively high private quality, \( b_I \geq 1/2 \), then the incumbent on an average expects a rival who is inferior to him in terms of private quality. If the incumbent raises \( x \) using nonvoting shares, thereby keeping constant the number of shares entitled to takeover premium, the incumbent makes the control contest relatively more dependent on
private benefits vis-á-vis dividends. Thus, the incumbent takes up all available positive NPV projects.

Proposition 10 does not imply that the incumbent with relatively high private quality, if allowed, will always choose nonvoting shares to fund the new investments. This is because nonvoting shares dilute dividends and entail a cost on the incumbent. It might very well be that an incumbent with \( b_I \geq \frac{1}{2} \), chooses voting shares when he can also choose nonvoting shares to fund the new investments.

If the incumbent has relatively high private quality, \( b_I \geq \frac{1}{2} \), and uses voting shares to fund the new investment opportunity, then two implications arise for the incumbent’s expected wealth: First, his expected private benefit can decrease because the likelihood that he retains control of the firm decreases as \( x \) increases. But new investment increases his share of the dividend, \( N \beta V_D^1 \), as well as his level of actual private benefit, \( b_I \alpha a_I P(x) \). Hence, the level he invests depends on the net effect of increasing \( x \) on his expected wealth. This result is formalized in the proposition below.

**Proposition 11.** If the incumbent manager uses voting shares to fund these new projects, then the necessary conditions for the incumbent manager to forgo some positive NPV projects are (i) \( \beta > 0 \) and (ii) \( b_I \geq \frac{1}{2} \).

*Proof.* See the proof stated in Section 8.4.

Even if the expected private benefit of the incumbent falls as he invests more using voting shares, the higher value of the dividend claim resulting from the increased investment can increase the incumbent’s expected wealth. Not only that, by our design, the level of private benefit also increases with the level of investment. Thus, if the negative impact of investment on the likelihood of retaining control outweighs the positive effects on dividends and private benefits, then only the incumbent will underinvest. Next, we derive the sufficiency condition for under-investment.

**Proposition 12.** When the incumbent manager is forced to use voting shares to fund these new projects and he owns some equity in the firm (\( \beta > 0 \)), then the incumbent manager forgoes some positive NPV projects if his private quality \( b_I \geq \hat{b}_I \), where

\[
\hat{b}_I = \min \left[ \frac{\alpha(2 - \alpha)\beta + \frac{2(1-\beta)^2 \log(1 + \frac{\alpha \beta}{1-\beta})}{\beta^2} - \frac{2\alpha(1-\beta)^2}{\beta(1-(1-\alpha)\beta)}}{\alpha^2 \left( \frac{2(1-\beta)}{1-(1-\alpha)\beta} - \frac{\alpha(2-\alpha)\beta^2}{1-\beta} \right)} \right], \]

*Proof.* See the proof stated in Section 8.5.

The condition provided in the Proposition 12 is a sufficient condition for under-investment. The manager will always forgo some positive NPV investment if he is forced to use voting shares to
fund the new investments and his private quality is greater than \( \hat{b}_I \). The manager may forgo some positive NPV projects for even smaller values of his private quality, \( \frac{1}{2} < b_I \leq \hat{b}_I \). We can interpret \( \hat{b}_I \) as a proxy for the likelihood of under-investment: Incumbent managers with \( b_I > \hat{b}_I \) are “sure candidates” for forgoing positive NPV projects. As \( \hat{b}_I \) gets bigger, it gets less likely that we will find an incumbent manager with private quality greater than \( \hat{b}_I \); hence, it becomes also less likely that the incumbent manager will underinvest.

A part of the cost of under-investment is borne by the manager because he owns equity in the firm. The larger his ownership, the larger the incumbent’s share of the costs of under-investment. Thus, the condition given in the Proposition 12 depends on \( \beta \). If the manager owns 10% of the equity in the firm and we assume quite effective other private benefit limiting mechanisms (\( \alpha = 0.30 \)), then the incumbent forgoes some positive NPV projects only if \( b_I > 0.795 \). This implies that if we collect a sample of firms with 10% manager’s equity ownership in countries with other reasonably effective monitoring mechanisms in place (as measured by \( \alpha = 0.3 \)), then we should expect to find some under-investment in 20.5% of the firms. Figure 4 depicts the likelihood that the incumbent will underinvest, \( 1 - \hat{b}_I \), as a function of an incumbent’s ownership fraction, \( \beta \), and ineffectiveness of other private benefit limiting mechanisms, \( \alpha \).

For any given value of \( \alpha \) as the incumbent’s ownership fraction, \( \beta \), increases, not surprisingly the likelihood of under-investment, \( 1 - \hat{b}_I \), decreases. As the incumbent’s ownership fraction increases, part of the costs of under-investment that the incumbent bears increase. Because the level of investment is the incumbent’s choice, he reduces the level of under-investment (or increases the level of investment) to maximize his objective function. For a given level of holding, \( \beta \), if the ineffectiveness of outside monitoring mechanisms, \( \alpha \), increases, then losing control of the firm becomes more costly for the incumbent; hence, he chooses an investment level to maximize the likelihood that he will retain control after the investments.

### 4.2 Welfare of Existing Shareholders and the Incumbent Manager

So far we have obtained conditions under which investment increases or decreases if voting shares or nonvoting shares are issued to raise funds. Increased investment financed by nonvoting equity is not always in the best interests of outside shareholders as well as the incumbent. There are costs to issuing nonvoting equity. These costs are considered here.

The value of the voting share is made up of two parts - the value of the dividend received and the value of the private benefits extracted in a takeover contest as a takeover premium. Because investors in nonvoting equity are not entitled to vote in a control contests, they do not receive the extracted private benefit. Therefore, investors are willing to pay a lower amount for nonvoting shares than for voting shares. This means that a larger number of nonvoting shares, \( n^0 \), relative to voting shares, \( n^1 \), have to be issued to finance a given level of investment, \( x \), reducing the per-share dividend that is available to owners of the existing voting shares. This is what is the “dividend dilution” effect.
Figure 4: This figure depicts the likelihood that the incumbent underinvests, $1 - \hat{b}_I$, as a function of incumbent’s equity holding, $\beta$, and ineffectiveness of other monitoring mechanism, $\alpha$. In panel A, we plot $\hat{b}_I$ as a function of $\beta$. We use two values of $\alpha$: 20% and 40%. For any given value of $\alpha$ as $\beta$ increases, $1 - \hat{b}_I$ approaches its minimum value of 0, implying that under-investment becomes less and less likely. In panel B, we plot $1 - \hat{b}_I$ as a function of $\alpha$. Again, we use two values of $\beta$: 5% and 15%. For any given $\beta$, as $\alpha$ increases $1 - \hat{b}_I$ increases, implying that under-investment becomes more and more likely.

The second potential cost arises when the issuance of nonvoting shares decreases the chance that the incumbent will lose control. This is the “entrerenchment” effect. The value of the vote depends on two factors – the private benefit extracted in a takeover and the probability of a takeover. Holding constant the private benefit extracted, a reduction in the probability of a successful takeover reduces the value of the voting rights. There is an offsetting gain. The voting rights remain concentrated in the hands of only the old shareholders. This leads to the sharing of the extracted private benefit among a smaller number of investors, increasing the per-share benefit.

To summarize, the issuance of nonvoting shares affects existing shareholders and the manager in three ways: (i) Lower per-share dividends; (ii) a lower probability of a change in control; and, (iii) a higher per-share takeover premium conditional on a takeover. Let us consider the value of the existing voting shares. Lower per-share dividends result in a lower public value for these shares. Similarly, the lower probability of a change in control results in a lower value for the vote, while a higher per-share takeover premium causes a higher value for the vote. Our first result obtains conditions under which the value of existing voting shares is higher if nonvoting shares are used to enhance the investment level. This proposition gives conditions under which existing shareholders will agree to fund new investments with nonvoting shares.

**Proposition 13.** For all $b_I \geq \hat{b}_I$ and $\bar{x}$ such that $n^0(\bar{x}) \leq N$, existing outside shareholders prefer investment financed using nonvoting shares if

$$1 - \frac{P(x)}{P(\bar{x})} \geq \frac{a_I^2 \alpha b_I (2 - \alpha)(2 + b_I(2 \alpha \beta + \alpha) - 2 \beta)}{2a_I(1 - \beta)^2 (1 - \alpha b_I) - (2 - \alpha)(a_I - a_I \beta(1 - \alpha b_I))^2 + (2 - \alpha)(1 - \beta)^2}.$$  

**Proof.** See the proof stated in Section 8.6. \qed
The result above gives the level of minimum under-investment needed before the existing shareholders are voluntarily willing to allow the incumbent to raise funds by issuing nonvoting shares. The LHS of the above inequality is a measure of the loss to shareholders because of under-investment, while the RHS is a measure of the costs related to the issuance of nonvoting shares. It is optimal for outside shareholders to allow the manager to issue nonvoting shares when the gains realized from reduced under-investment outweigh the costs. The level of under-investment needed for outside shareholders to prefer nonvoting shares is quite small. If \( a_I = 0.5 \), \( b_I = 0.73 > 0.726 = \hat{b}_I \), \( \alpha = 0.2 \), and \( \beta = 0.05 \), outside shareholders will find the issuance of nonvoting shares to finance an investment optimal, even if under-investment is just about 6.8%. Similarly, if \( a_I = 0.5 \), \( b_I = 0.8 > 0.795 = \hat{b}_I \), \( \alpha = 0.3 \), and \( \beta = 0.1 \), outside shareholders will find nonvoting shares optimal if under-investment is close to 13%.

Next, we consider the incumbent’s expected wealth. Use of nonvoting shares to fund the new investments lowers the per-share dividend and thus affects the incumbent’s wealth negatively. If \( b_I \geq 1/2 \), then nonvoting shares lower the probability of a successful takeover and thus helps to increase expected wealth of the incumbent.\(^36\) The propositions below provide results on the types of managers who are better off if the firm issues nonvoting stock.

**Proposition 14.** For all \( 1/2 \leq b_I \leq 1 \) and \( x \) such that \( n^0(x) \leq N \), the incumbent prefers investment financed by nonvoting equity if \( b_I \geq \hat{b}_I \), where

\[
\hat{b}_I = \frac{2(1 - \beta) \left( (2 - \beta) \left( \alpha(2 - \alpha) \beta^2 + (1 - \beta) \log \left( 1 + \frac{\alpha \beta}{1 - \beta} \right) \right) - (1 - \beta) \beta \log \left( 1 + \alpha + \frac{\alpha \beta}{1 - \beta} \right) \right)}{4 \alpha(2 - \beta) \beta \left( (1 - \beta) \log \left( 1 + \alpha + \frac{\alpha \beta}{1 - \beta} \right) - (1 - \beta) \log \left( 1 + \frac{\alpha \beta}{1 - \beta} \right) - \alpha(2 - \alpha) \beta \right)}.
\]

**Proof.** See the proof stated in Section 8.7. \( \square \)

From equation (24) we see that investment using nonvoting shares increases the likelihood that the incumbent retains control of the firm if \( b_I \geq \frac{1}{2} \). From Proposition 12 we know that the incumbent is better off if nonvoting shares can be issued and \( b_I \geq \hat{b}_I > 1/2 \); that is, the minimum value of \( b_I \) in Proposition 12 is higher than \( 1/2 \). The divergence exists because of the cost related to dividend dilution associated with nonvoting shares. If nonvoting shares instead of voting shares are issued, then the aggregate dividend is divided among \( N + n^0 \) shareholders, which is strictly greater than \( N + n^1 \) claims if voting shares are issued. If \( \beta = 0.1 \) and \( \alpha = 0.3 \), the manager prefers nonvoting shares if \( b_I \) is greater than 0.61, even though the incumbent’s likelihood of retaining control increases with investment for all \( b_I \geq 0.5 \). For all \( \beta > 0.185 \) given \( \alpha = 0.3 \), \( \hat{b}_I \geq 1 \), implying that the incumbent never likes nonvoting shares.

From Propositions 11, 13, and 14 we can derive some interesting observations. Consider the case when \( b_I \in [1/2, 1] \). It is easy to show that for all \( \beta \) and \( \alpha \) such that \( \hat{b}_I \) and \( \hat{b}_I \) are within the range \( 1/2 \) and 1, \( \hat{b}_I > \hat{b}_I \). We depict the comparable values of \( \hat{b}_I \) and \( \hat{b}_I \) in Figure 5 below.

\(^36\) The increase in the takeover premium does not affect the manager because we have assumed that he does not tender in a takeover.
Figure 5: This figure depicts $\hat{b}_I$ and $\hat{\hat{b}}_I$ as a function of incumbent’s equity holding, $\beta$. We set $\alpha = 0.5$. For any given value of $\alpha$, as $\beta$ increases both $\hat{b}_I$ and $\hat{\hat{b}}_I$ approach 1, implying that under-investment becomes less likely, just as the manger is less likely to choose nonvoting shares.

Thus, we can divide the range of the incumbent’s private qualities into four parts: $[0, 1/2)$, $[1/2, \hat{b}_I)$, $[\hat{b}_I, \hat{\hat{b}}_I)$, and $[\hat{\hat{b}}_I, 1]$. If the incumbent is forced to use only voting shares, then the incumbent with $b_I \in [\hat{b}_I, 1]$ underinvests. Otherwise, he invests in all available positive NPV projects. If the incumbent is given a choice of using either voting shares or nonvoting shares, then the incumbent always invests in all available positive NPV projects. The manager will use voting shares to fund all investments, if his private quality $b_I \in [0, \hat{\hat{b}}_I)$. The manager will use nonvoting shares to fund all investments, if his private quality $b_I \in [\hat{\hat{b}}_I, 1]$. Figure 7 below depicts these $b_I$ ranges.

Allowed to use either voting shares or nonvoting shares

- Incumbent uses voting shares
  - $b_I = 0$
  - $b_I = 1/2$
  - $b_I = \hat{b}_I$
  - $b_I = \hat{\hat{b}}_I$
  - $b_I = 1$

- Incumbent uses nonvoting shares

- Incumbent invests $\bar{x}$.

Forced to use only voting shares

- $b_I = 0$
- $b_I = 1/2$
- $b_I = \hat{b}_I$
- $b_I = \hat{\hat{b}}_I$
- $b_I = 1$

- Incumbent invests $\bar{x}$.
- Indeterminate
- Incumbent invests $x < \bar{x}$.

The final question that we answer in this subsection is in what instances we will observe firms

---

$^{37}$If the incumbent’s $b_I \in [\hat{b}_I, \hat{\hat{b}}_I]$, he may or may not under invest. This range is indeterminate as we are only able to solve sufficient condition of under-investment as well as incumbent's choice of voting vs. nonvoting shares.
issuing nonvoting shares. The answer depends on the balance of power between the manager and the shareholders. If shareholders have the upper hand and can force the manager to issue a particular type of security, the condition given in Proposition 11 will determine when the firm will issue nonvoting shares. In contrast, if shareholders can only specify a menu of securities, the conditions in Proposition 13 and Proposition 14 will both have to be satisfied before the firm issues nonvoting shares.

4.2.1 Low Quality Managers and the Control Contest

Next, we turn to the issue of economic efficiency. Other studies have found that dual-class shares allow control of the firm to remain in or pass to the hands of inferior managers, lowering economic efficiency. We show that it is true that dual-class shares allow inferior managers to win control contests. A statement on economic efficiency, though, requires analysis of a trade-off between the costs of under-investment and the cost of inefficiently managed firms. This process requires assumptions regarding the ability of other firms to undertake projects that the firm under consideration has forgone. We leave this aspect of the problem to future research.

Grossman and Hart (1988) show that voting shares are optimal because they ensure that the firm ends up in the hands of the manager who is of high public quality. Our result is similar to their result. The difference between the voting shares and the nonvoting shares is that nonvoting shares cause the private quality of managers to have a larger impact on the outcome of the control contest. Consider a rival with private quality higher than the incumbent’s private quality, that is, \( b_R > b_I \). Nonvoting shares favor this rival in a control contest, making it easier for him to gain control of the firm; that is, he can gain control for lower values of \( a_R \), values for which he would lose the control contest if the firm had financed its investment using voting shares. Similarly, if \( b_R < b_I \); that is, if the incumbent has a higher private quality, nonvoting shares would favor the incumbent in a control contest, making it easier for him to retain control of the firm. That is, an incumbent can keep control of the firm for lower levels of \( a_I \), values for which he would lose control of the firm if the investment had been financed using voting shares. The proposition below formalizes this result.

**Proposition 15.** The minimum public quality required for an incumbent manager to retain control of the firm is lower in firms financed with dual-class shares.

*Proof.* See the proof stated in Section 8.8.

The fact that a manager of lower public quality may gain control of firms will be an important consideration for market regulators. If other mechanisms can be used to discipline managers, the cost of this problem will be small. Moyer et al. (1992) find that alternative monitoring mechanisms emerge in firms after the issuance of dual-class shares.\(^{38}\) This is an additional issue that will make the costs and benefits of dual-class shares difficult to evaluate.

\(^{38}\)Hollinger International presents a good example of the negative effects of dual-class shares. Former CEO Conrad Black controlled all of the company’s class-B shares, which gave him 30% of the equity and 73% of the
The next section discusses extensions to our model. It also looks at the effect of relaxing some of our initial assumptions.

5 Extensions

In this section we consider three related issues. First we allow the incumbent to tender his holdings in a control contest. This is important because it helps to make the incumbent more entrenched. Second we address the issuance of shares with fewer than one vote per-share. The reason that these may be useful is that they would have lower costs related to dividend dilution than zero-vote shares. Firms in Japan and the U.S. are allowed to issue multiple classes of shares. Third, we discuss the costs and benefits of multiple classes of shares.

5.1 Entrenchment and Investment when the Incumbent Tenders

A change in control occurs when the rival manager can offer a higher per-share value to the outside shareholders than the incumbent. In our initial setup we assumed that the incumbent does not tender his shares in the control contest. We now relax this assumption and allow the manager to tender. If the manager tenders his shares in a control contest, then the rival’s private benefit is divided over a larger number of shares, $N + n$ as opposed to $(1 - \beta)N + n$ outside voting shares when the manager is not able to tender. This puts the rival at a disadvantage relative to the incumbent. The probability of a takeover can be obtained by considering the cases of nonvoting shares and voting shares separately. The incumbent can retain control if he can offer more for the shares than the rival. If voting shares are used to finance the investment, this contest is equivalent to

$$\frac{x + a_I P(x)}{N + n^1} - \frac{b_I \alpha a_I P(x)}{(1 - \beta)N + n^1} + \frac{b_I \alpha a_I P(x)}{N + n^1} \geq \frac{x + a_R P(x)}{N + n^1} - \frac{b_R \alpha a_R P(x)}{N + n^1} + \frac{b_R \alpha a_R P(x)}{N + n^1}. \quad (30)$$

The first two terms on the LHS of equation (10) are the per-share public value that is generated with the incumbent in control. The third term on the LHS is related to the incumbent’s private value. The denominator is smaller here than in the first two terms because the private benefit is distributed only to the outside shareholders. The RHS terms are related to the public and private benefit per-share generated under the rival. The private benefits of the potential rival voting power. He ran the company as if he were the sole owner, exacting huge management fees, consulting payments, and personal dividends. Hollinger’s board of directors was filled with Black’s friends who were unlikely to forcefully oppose his authority. Holders of publicly traded shares of Hollinger had almost no power to make any decisions in terms of executive compensation, mergers and acquisitions, board construction poison pills, or anything else for that matter. Hollinger’s financial and share performance suffered under Black’s control.

39See, for example, Burkart et al. (1998) for more discussions.
is divided among all the shareholders of the firm. Simplifying equation (30) we obtain

\[ a_I \left(1 + \alpha \kappa^1 b_I \right) \geq a_R, \]  

where \( \kappa^1 = \frac{N \beta}{(1 - \beta) N + n} \). From equation (15) we know that \( \bar{a}_R = a_I \left(1 + \alpha \kappa^1 b_I \right) \). Thus, the incumbent retains control of the firm if \( a_R \in [0, \bar{a}_R] \). The private quality of the potential rival plays no role in the control contest, whereas the private quality of the incumbent plays a significant role in the control contest. When the incumbent can tender, the range of the potential rival’s qualities over which the incumbent retains certain control, \([0, \bar{a}_R] \), is much higher than \([0, \bar{a}_R^1] \), the range when the incumbent does not tender. Since the range over which the incumbent certainly loses control, \([\bar{a}_R, 1]\), remains the same, the likelihood that the incumbent retains control of the firm after investing \( x \), say \( \hat{\phi}^1(x) \), is greater than \( \phi^1(x) \), the likelihood that the incumbent retains control of the firm after investing \( x \), when the incumbent is not able to tender.

What happens to the incumbent’s likelihood of retaining control of the firm if \( x \) increases? Because \( \frac{\partial \kappa^1}{\partial x} \) is negative, \( \frac{\partial \hat{\phi}^1}{\partial x} \) is also negative. Hence, the incumbent’s likelihood of retaining firm control decreases as the incumbent raises the level of investment.

If nonvoting shares are issued to finance the investment and if the manager can tender his shares in a control contest, the rival’s private benefit is divided over a larger number of shares, \( N \) rather than \( (1 - \beta)N \). Then the incumbent retains control if

\[ \frac{x + a_I P(x)}{N + n^0} - \frac{b_I \alpha a_I P(x)}{(1 - \beta) N} \geq \frac{x + a_R P(x)}{N + n^0} - \frac{b_R \alpha a_R P(x)}{N} \tag{32} \]

The private value is distributed to all shareholders who own voting shares. The holders of the nonvoting shares do not get a share of the private value because they cannot affect the outcome of the control contest. Simplifying equation (32) we get

\[ a_I \left(1 + \alpha \kappa^0 b_I \right) \geq a_R \left(1 + \alpha \kappa^0 b_R \right), \]  

where \( \kappa^0 = \frac{N \beta + n^0}{(1 - \beta) N} \) and \( \kappa^0 = \frac{n^0}{N} \). Also note that \( \kappa^0 - \kappa^0 = \frac{\beta(N + n^0)}{(1 - \beta) N} > 0 \). This gives

\[ \hat{a}_R^0 = a_I (1 + \alpha \kappa^0 b_I) = \bar{a}_R^0 \quad \text{and} \quad \hat{a}_R^0 = \frac{a_I (1 + \alpha \kappa^0 b_I)}{1 + \alpha \kappa^0} = \frac{\bar{a}_R^0}{1 + \alpha \kappa^0} > a_R^0. \]  

From equation (34), we see that the range of the potential rival’s qualities over which the incumbent retains certain control, \([0, \hat{a}_R^0]\), is larger than \([0, \bar{a}_R^0]\), rival’s qualities over which the incumbent retains control when the incumbent is not able to tender. Since the range over which the incumbent loses control, \([\hat{a}_R^0, 1]\), remains the same in this case also, the incumbent’s likelihood of retaining control of the firm for a given level of investment, \( \hat{\phi}^0(x) \), is at least weakly greater when the manager can tender, relative to the likelihood of retaining control of the firm.
for the same level of investment, \( \phi^0(x) \), when the manager cannot tender. This is formally stated in the proposition below.

**Proposition 16.** Irrespective of the type of financing, voting or nonvoting, private quality of the incumbent, \( b_I \), plays a relatively more decisive role in the control contest and makes the incumbent more entrenched, if the incumbent can tender his shares in the control contest.

**Proof.** Follows directly from equations (31) or (34).

Like our initial setup, these bounds determine the outcome of the control contest: Likelihood that the incumbent retains control of the firm after investment depends on the level of investment, and the incumbent can choose the size of \( x \). Because the number of new shares needed to finance the investment increases as the size of the investment increases, \( \frac{\partial n^0}{\partial x} > 0 \), then \( \frac{\partial \hat{\kappa}^0}{\partial x} > 0 \). Thus,

\[
\frac{\partial}{\partial x} \hat{\omega}_R = \frac{\partial}{\partial n^0} \hat{\omega}_R \times \frac{\partial n^0}{\partial x} = -\frac{a_I N \alpha (1 - \beta - \frac{b_I}{2}(1 - \alpha \beta))}{(N + n^0 \alpha)^2(1 - \beta)} \times \frac{\partial n^0}{\partial x} < 0.
\]

(35)

Thus, the implications of increasing investments on the incumbent manager’s likelihood of retaining control of the firm remains qualitatively similar to the case when the manager is not able to tender.

5.2 Optimal vote-dividend combination

The optimal vote-dividend combination can be thought of in two different ways. The first is to consider shares that have 1 unit of dividend and \( \theta \) votes, and to find the optimal value of \( \theta \). The second is to allow the firm to simultaneously issue both voting and nonvoting shares. We first consider \( \theta \)-vote shares.

The optimality of \( \theta \)-vote shares, with \( 0 < \theta < 1 \), will depend on the size of the investment opportunity available to the firm. For a class of shares, the vote will have value only if a sufficient mass of votes of that class exist so that these shares can be used by the manager to block a takeover. This means that managers will issue \( \theta \)-vote shares only if the investment opportunity is large enough that \( \beta N + n \theta \geq \frac{1}{2} (N + n \theta) \).

The reasoning that is captured by the above inequality is as follows: Consider a firm with two classes of shares, one-vote and \( \theta \)-vote, outstanding. Suppose \( n \theta \) is small so that the above inequality is not fulfilled. In this case the manager has no incentive to bid for this \( \theta \)-vote class of shares; blocking the rival requires the manager to bid for the voting shares. The rival has no incentive to bid for the \( \theta \)-vote shares either. The outcome of the control contest is determined solely by the owners of the voting shares. This causes the vote to have zero value in the \( \theta \)-vote shares, giving the manager no incentive to issue \( \theta \)-vote shares.

If \( \bar{x} \) is small, \( \theta = 0 \) is likely to be optimal. This is because the number of shares that are issued is going to be small for small \( \bar{x} \), and the total number of votes held by shareholders
in that class will be insufficient to meet the above condition. Our model has assumed that shareholders are homogeneous. Heterogeneity among shareholders may result in cases where \( \theta \)-vote shares may become optimal even when \( \bar{x} \) is small.

Allowing firms to simultaneously issue both nonvoting and voting shares will increase the set of firms that find it optimal to issue dual-class shares. This assertion is based on the following line of reasoning: Existing one-vote shareholders prefer nonvoting shares when the level of under-investment is high; that is, \( 1 - \frac{P(x)}{P(\bar{x})} \geq g(\alpha, \beta, \kappa^0, b_I, a_I) \). From equation (P13-5) we know that the RHS of the above inequality is an increasing function of \( \kappa^0 \) which itself is an increasing function of \( n^0(\bar{x}) \). Hence, as \( n^0(\bar{x}) \) decreases, the outside shareholders will find it optimal to allow the manager to finance investments using nonvoting shares, even for low levels of under-investment. If the investments are partly financed using voting shares and this analysis is carried out over the remaining projects, the relevant \( n^0(\bar{x}) \) will have a smaller value, implying that the existing shareholders, the owners of the voting shares, would be more willing to allow managers to issue nonvoting shares. In this case the existing shareholders could allow the manager the choice of issuing voting shares or a mix of \( \theta \)-voting shares per nonvoting share issued.

### 5.3 Multiple classes of shares

We considered a firm that issues only two classes of shares: Voting and nonvoting shares. One logical extension to this model is to consider multiple classes of shares. Is it optimal either for the manager or for existing shareholders to issue multiple classes of shares? Shares that give their owners fractional voting rights can be considered. Thus, the firm could simultaneously issue shares with \( \theta_0, \theta_1, \theta_2, \) and \( \theta_3 \) votes (an example can be \( \theta_0 = 0, \theta_1 = 0.33, \theta_2 = 0.5, \) and \( \theta_3 = 1 \)). In the above framework, the shares with fractional votes will be issued if the fractional votes have value. The fractional votes will have value if there is a sufficient mass of each of these classes of shares outstanding so that the rival is forced to buy them to take control of the firm.

The manager can raise the cost of a takeover for the rival by issuing multiple classes of shares. This does not mean that it is optimal for the manager to issue multiple classes of shares. The manager bears a cost when he issues multiple classes of shares. This cost is in the form of lower dividends. The existing shareholders are likely to find multiple classes of shares detrimental to their interest. As the number of classes of shares increases the probability of a change in control is likely to decrease very quickly. The compensating factor, investment, is unlikely to go up fast enough to increase the value of the shares held by outside shareholders. Thus, multiple classes of shares are unlikely to be optimal for old shareholders.
6 Conclusions

We analyze the decision problem of a firm with a set of available positive NPV projects. We show that if a firm requires outside equity financing to undertake profitable investment projects, then in some cases managers will find separation of the voting and dividend claims optimal. Raising equity capital has two effects: (i) The value of the firm increases as positive NPV projects are undertaken and (ii) the proportion of the firm’s shares owned by the manager decreases, increasing the likelihood that the manager loses control of the firm. Thus, a manager who values control that arises from the private benefits it provides, will find it optimal to forgo some positive NPV projects under these circumstances as long as the project’s NPV is not too high. A pure dividend claim enables the manager to finance the investment without diluting his control, and thus increasing his chances of losing control of the firm, which in turn increases the manager’s willingness to undertake all positive NPV projects.

This study provides a theoretical justification for easing the regulations on the issuance of dual-class shares, which has recently been proposed or enacted in a number of developed and developing countries. This theory also allows us to explain the results of the empirical studies that find a positive abnormal return to a firm’s announcement that it is issuing a second class of inferior voting rights shares. In our model, the announcement of dual-class recapitalizations would indicate to shareholders a reduction in the severity of under-investment problem in the firm, which should trigger an increase in the value of the firms shares.

Our theoretical results are consistent with empirical studies such as Lehn et al. (1990) and Dimitrov and Jain (2006), which predict firms with greater growth (or investment) opportunities are more likely to undertake dual-class recapitalizations. Also, our theory is supported by the empirical findings of Faccio and Masulis (2005), who indicate a reluctance on the part of controlling shareholders to issue voting equity to finance M&A activities owing to a fear of loss of control. Hence, cash-strapped firms will forgo positive NPV acquisition opportunities to prevent dilution of the manager’s control rights. In addition, our theory is consistent with studies by Nenova (2003) and Smart and Zutter (2003) that find dual-class structures make takeovers less likely but conditional on an actual takeover occurring, takeover premiums paid to the outside shareholders are higher.\textsuperscript{40} Our model produces new empirical predictions regarding the relationships among incumbent management quality, management ownership, and the effectiveness of other mechanisms to prevent the extraction of private benefits by the incumbent manager and the likelihood of dual-class recapitalization. We show outside shareholders are more likely to allow dual-class structures if the incumbent manager has an intermediate level of ownership and a relatively high ability to extract private benefits of control, and if the operating environment of the firm is such that other mechanisms to reign in private benefits

\textsuperscript{40}Krishnan and Masulis (2011) show that more reputable target M&A legal advisors are associated with a lower probability of takeovers, but a higher takeover premium, where the percentage fall in the probability is more than offset by a higher percentage increase in the takeover premium.
are ineffective or overly costly. This is because these conditions imply the manager’s extraction of high private benefits, with a low share of the cost of under-investment. As a consequence, there is a high likelihood of value-destroying under-investment if the manager is constrained only to use voting shares to finance these new investments.

References


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7 Appendix A

7.1 A Numerical Example

Consider a firm that has a public value of $2.00, generates a private value for the incumbent of $0.20 and has 100 shares outstanding. The value of the existing firm, both public and private, is the same under both the incumbent and the rival manager. The incumbent owns 50 shares in the firm and the incumbent is wealth constrained. Given our assumption that the incumbent owns half the shares in the firm, there is a zero probability of a change in control of the firm, \( \phi = 1 \), without the incumbent’s consent.
The expected value of the incumbent’s stake in the firm is the sum of the expected public value of the shares that he owns, plus the expected private benefits of control; that is, the value of the incumbent’s stake in the firm is $1.20 (\(= \frac{1}{2} \times $2.00 + \phi \times 0.20 = $1.00 + 1.0 \times 0.2\)). The value of the shares owned by outside shareholders is the probability of the incumbent’s retaining control times the public value of the firm under the incumbent, plus the probability of the rival’s gaining control times the price paid by the rival. Thus, the value of the shares owned by the existing outside shareholders is $1.00.

To keep the numerical example simple, we assume that the incumbent has to choose from three discrete investment levels: invest nothing, invest $1.00, or invest $2.00. If the incumbent invests nothing, there is no addition to the value of the firm and no new shares are issued. If the incumbent invest $1.00 or $2.00, the resulting value of the firm, additional public and private value generated under the incumbent and a rival manager are summarized in Table I below.

<table>
<thead>
<tr>
<th>Existing Firm Value</th>
<th>Investment Opportunity</th>
<th>Number of New Shares Issued</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public Value</td>
<td>Private Value</td>
<td>Public Value</td>
</tr>
<tr>
<td>2.00</td>
<td>0.20</td>
<td>0.00</td>
</tr>
<tr>
<td>2.00</td>
<td>0.20</td>
<td>1.00</td>
</tr>
<tr>
<td>2.00</td>
<td>0.20</td>
<td>2.00</td>
</tr>
</tbody>
</table>

Figure 6: This table summarizes the value of the existing firm and the additional private and public value generated by the new investment opportunity and under the incumbent and the rival manager. The first row corresponds to the situation where the incumbent management does not undertake any new investment. The second and third rows correspond to the cases where the incumbent invests $1.00 and $2.00, respectively. Investment in the projects adds to the public value of the firm and to the private benefits of the manager-in-control.

Investment in the projects adds to the public value of the firm and also to the private benefits of the manager-in-control at the end of the investment horizon. We assume that the rival manager is strictly better than the incumbent: The rival manager can generate public value that is higher than the sum of the public and private value that the incumbent can generate. For example, if the incumbent invests $1.00 and he is the manager-in-control at the end of the investment horizon, then the public value is $1.10 and his private benefit is $0.06, giving an aggregate value $1.16. Whereas, if the incumbent invests $1.00 and the rival
The expected value of the incumbent’s stake in the firm is the sum of the expected public value of the shares that he owns plus the expected private benefits of control. The expected public value of a share in the firm is the probability that the incumbent retains control times the public value of the firm under the incumbent, plus the probability that the rival manager gains control times the public value of the firm under the rival manager. For investment level of $1 the expected public value is equal to the expected NPV under the incumbent plus the expected NPV under the rival manager; that is, $2 + (0.95 \times (1.1 - 1) + 0.05 \times (1.18 - 1)) = 2.104. The expected private benefit extracted by the incumbent is the private benefit of control times the probability of remaining in control. For investment level of $1.00 the expected private benefit is
0.95 × 0.26 or 0.247. Therefore, the expected value of the incumbent’s stake if he invests $1.00 is $0.5 \times (2.104) + 0.247$ or 1.299. For the investment level $1.00, the shareholders’ expected wealth is $0.5 \times 2.104$ or 1.052. If the incumbent has a choice regarding the type of equity to issue to finance the project, the incumbent will issue nonvoting equity to invest $1.00 and the expected value of the shares owned by existing outside shareholders is $1.05$. For investment level of $2.00, the expected public value of the firm is $2 + 0.79 \times (2.12 - 2) + 0.21 \times (2.20 - 2)$ or 2.1368.

The expected private benefit extracted by the incumbent is $0.79 \times 0.27$ or 0.2133. Therefore, the expected value of the incumbents stake for investment level of $2.00$ is $0.5 \times 2.1368 + 0.2133$ or 1.2817. The expected welfare of the outside shareholders is $0.5 \times 2.1368$ or 1.0684. Table III summarizes this information.

Table III

<table>
<thead>
<tr>
<th>Investments</th>
<th>Voting Shares Issued to Finance New Investment</th>
<th>Nonvoting Shares Issued to Finance New Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Manager’s Payoff</td>
<td>Outside Shareholders’ Payoff</td>
</tr>
<tr>
<td>0.00</td>
<td>1.2000</td>
<td>1.0000</td>
</tr>
<tr>
<td>1.00</td>
<td>1.2990</td>
<td>1.0520</td>
</tr>
<tr>
<td>2.00</td>
<td>1.2817</td>
<td>1.0684</td>
</tr>
</tbody>
</table>

Figure 8: The first half of the table shows the expected payoff of the incumbent and outside shareholders when voting shares are issued to finance the investment, and the second half of the table shows payoff of the incumbent and outside shareholders when nonvoting shares are issued to finance the investment. Outside shareholders always want the manager to invests in all positive NPV projects. But if forced to use voting shares, then it is optimal for the incumbent to invest $1.00 rather than $2.00 given his payoff from investing $1.00 is $1.2990 which is strictly greater than $1.2817 – incumbent’s payoff from investing $2.00.

The incumbent’s expected wealth is maximized at an investment level of $1.00 when the investment is finance using voting shares and at an investment level of $2.00 when the investment is financed using non-voting shares. The decision made by the existing outside shareholders is related to the type of shares that the firm can issue to finance the new investment. If the incumbent is required to finance the investment by issuing voting shares, the incumbent will invest $1.00 and the expected value of the shares owned by existing outside shareholders is $1.052. From the table it can be seen that there are situations in which it is value increasing for outside shareholders to allow the incumbent to issue nonvoting shares to finance investments.
This increases the outside shareholders’ wealth from $1.052 to $1.06. This is true regardless of the fact that nonvoting shares are likely to entrench the incumbent and prevent better rivals from taking over the firm. The difference in the value of the shares owned by the existing outside shareholders when voting and nonvoting shares are used to finance the investment is a cost of entrenchment (for investment level $1, the costs entrenchment is $1.052 − $1.05 = $0.002) per dollar of investment.

Allowing the manager to issue nonvoting shares will raise the value of the shares owned by existing outside shareholders when the loss in value from under-investment is larger than the loss in value from entrenchment. Examples of this situation are firms that have many growth opportunities and firms in relatively new industries. For firms that have relatively few growth opportunities the above result is unlikely to hold. In these firms under-investment is less likely to be a problem and will have a smaller negative impact on the value of the firm.

Does a contractual solution to the under-investment problem work? Often it may be possible to make a side payment to the manager to induce him to undertake the investment. This alternative would require the outside shareholders to compensate the manager for the decrease in expected wealth associated with an investment of $2 financed using one-vote shares. In the case presented above, contractual solution does not work. Increase in the outside shareholders expected wealth, $1.0684 − $1.052 = $0.0164, if investment level goes up from $1.00 to $2.00, is smaller than the drop in the incumbent’s expected wealth, $1.2817 − $1.299 = −$0.0173. Hence, cross-subsidization will not be feasible.

8 Appendix B

8.1 Basic Results

Before we present the proofs of the propositions, we first show some basic results that we will use repeatedly. By definition we have

\[ \kappa_0(x) = \frac{\beta N + r^0(x)}{(1 - \beta)N} \quad \text{and} \quad \kappa_1(x) = \frac{\beta N}{(1 - \beta)N + n^1(x)}, \]  

(BR-1)
where \(n_j(x) = 0\) at \(x = 0\), and \(n_j(x) > 0\), \(\forall x \in (0, \bar{x}]\), and \(j = 0, 1\). Also, \(\frac{\partial n_j(x)}{\partial x} > 0\). Thus,

\[
\kappa^0(0) = \kappa^1(0) = \frac{\beta}{1 - \beta}.
\]

(BR-2)

For \(x > 0\) we get

\[
\kappa^0(x) = \frac{\beta}{1 - \beta} + \frac{n^0(x)}{(1 - \beta)N} > \frac{\beta}{1 - \beta} = \kappa^0(0),
\]

(BR-3)

\[
\kappa^1(x) = \frac{\beta}{1 - \beta} - \frac{\beta n^1(x)}{(1 - \beta)N((1 - \beta)N + n^1)} < \frac{\beta}{1 - \beta} = \kappa^1(0).
\]

(BR-4)

Thus \(\kappa^0(x) \geq \kappa^1(x)\) \(\forall x \in [0, \bar{x}]\). Differentiating \(\kappa^0(x)\) and \(\kappa^1(x)\) with respect to \(x\) we get

\[
\frac{\partial \kappa^0}{\partial x} = \frac{\partial}{\partial x} \left( \frac{N \beta + n^0}{(1 - \beta)N} \right) = \frac{1}{(1 - \beta)N} \frac{\partial n^0}{\partial x} > 0.
\]

(BR-5)

and

\[
\frac{\partial \kappa^1}{\partial x} = \frac{\partial}{\partial x} \left( \frac{N \beta}{(1 - \beta)N + n^1} \right) = \frac{-1}{((1 - \beta)N + n^1)^2} \frac{\partial n^1}{\partial x} < 0.
\]

(BR-6)

**8.1.1 Dividend claims**

We can simplify equation (25) and rewrite as

\[
(N + n^j)V_D^j = x + \phi^j \times a_I(1 - \alpha b_I)P(x) + \int_{\bar{\sigma}_R^j}^{\pi_R^j} \int_{\bar{b}_R^j}^{b_R} (a_R(1 - \alpha b_R)P(x)) \, db_R \, da_R
\]

\[
+ \int_{\bar{\sigma}_R^j}^{\pi_R^j} \int_{0}^{1} (a_R(1 - \alpha b_R)P(x)) \, db_R \, da_R.
\]

(BR-7)

Integrating the third term of the above expression we get

\[
\int_{\bar{\sigma}_R^j}^{\pi_R^j} \int_{0}^{1} (a_R(1 - \alpha b_R)P(x)) \, db_R \, da_R = \frac{2 - \alpha}{4} \left( 1 - (\bar{\sigma}_R^j)^2 \right) P(x).
\]

(BR-8)

Integrating the second term of the above expression we get

\[
\int_{\bar{\sigma}_R^j}^{\pi_R^j} \int_{\bar{b}_R^j}^{b_R} (a_R(1 - \alpha b_R)P(x)) \, db_R \, da_R = A(x, \kappa^j) + B(x, \kappa^j),
\]

(BR-9)

where \(A(x, \kappa^j) = \frac{(\alpha R)^2(2+(4-5\alpha)\kappa^j+\alpha(6-\alpha)(\kappa^j)^2)P(x)}{4\alpha^2(1+\alpha\kappa^j)}\) and \(B(x, \kappa^j) = \frac{\alpha R^2 P(x)}{2\alpha \kappa^j} \ln(1 + \alpha \kappa^j)\).
8.1.2 Value of existing voting shares when voting shares are issued

Using equations (25) and (28) we can express total value of all voting shares when voting shares are issued to finance the new projects:

\[
(N + n^1)V^1_1 = (N + n^1)V^1_D + (N + n^1)V^1_{\text{vote}}
\]  
(BR-10)

\[
(N + n^1)V^1_1 = \int_0^{\pi_R^1} \int_0^1 FV I db_R da_R + \int_0^{\pi_R^1} \int_0^{\pi_R^1} FV_R db_R da_R + \int_0^{\pi_R^1} \int_0^{\pi_R^1} (N + n^1)B_I (1 - \beta)N + n^1 db_R da_R
\]

\[
= x + \frac{1}{2} P(x) \left(\alpha I (1 - \alpha b_I) + (1 + \kappa^1) \left(1 - \frac{\ln(1 + \alpha \kappa^1)}{\alpha \kappa^1}\right) \bar{a}^1_R + (1 - \alpha/2) \left(1 - (\bar{a}^1_R)^2\right)\right).
\]  
(BR-11)

If the firm issues voting stock to finance the investment, and because all new securities are issued at zero expected profit, \(n^1 V^1_1 = x\), we get

\[
NV^1_1 = \frac{1}{2} P(x) \left(\alpha I (1 - \alpha b_I) + (1 + \kappa^1) \left(1 - \frac{\ln(1 + \alpha \kappa^1)}{\alpha \kappa^1}\right) \bar{a}^1_R + (1 - \alpha/2) \left(1 - (\bar{a}^1_R)^2\right)\right).
\]  
(BR-12)

8.1.3 Value of existing voting shares when nonvoting shares are issued

If nonvoting shares are used to finance the investment, then using equations (25) and (28), the value of an existing voting share is

\[
(N + n^0)V^0_1 = (N + n^0)V^0_D + (N + n^0)V^0_{\text{vote}}
\]

\[
= \frac{P(x)}{2} \left(\alpha I (1 - \alpha b_I) + (1 + \kappa^0) \left(1 - \frac{\ln(1 + \alpha \kappa^0)}{\alpha \kappa^0}\right) \bar{a}^0_R + (1 - \alpha/2) \left(1 - (\bar{a}^0_R)^2\right)\right).
\]  
(BR-13)

8.2 Proof of Proposition 9

The manager chooses the investment level to maximize his objective function. We show that the first derivative of the manager’s objective function evaluated at \(\bar{x}\) is nonnegative, which implies that the incumbent invests \(\bar{x}\).

When investments financed using voting shares and \(\beta = 0\), then from equation (15) we get \(a^1_R = \bar{a}^1_R = a_I\). This is because \(\kappa^1 = \frac{\beta N}{(1 - \beta)N + n^1} = 0\). Thus,

\[
\phi^1(x) = \int_0^{a^1_I} da_R + \int_0^{a^1_I} \int_0^{\bar{a}^1_R} db_R da_R = \int_0^{a_I} da_R = a_I.
\]  
(P9-1)
Incumbent manager’s objective function if he finances the new investment issuing j-type shares, say $MO^j$, is

$$MO^j(x) = \beta NV_D^j(x) + \phi^j(x) b_I a_I P(x).$$  \hspace{1cm} (P9-2)

Differentiating the incumbent’s objective function with respect to $x$ gives

$$\frac{\partial MO^j(x)}{\partial x} = \beta N \frac{\partial V_D^j(x)}{\partial x} + b_I a_I \left( \frac{\partial \phi^j(x)}{\partial x} P(x) + \phi^j(x) \frac{\partial P(x)}{\partial x} \right). \hspace{1cm} (P9-3)$$

Substituting $j = 1$ and $\beta = 0$ we get

$$\frac{\partial MO^1(x)}{\partial x} = b_I a_I \left( \frac{\partial \phi^1(x)}{\partial x} P(x) + \phi^1(x) \frac{\partial P(x)}{\partial x} \right). \hspace{1cm} (P9-4)$$

But $\frac{\partial \phi^1(x)}{\partial x} |_{\beta=0} = \frac{\partial a_I}{\partial x} = 0$. Hence,

$$\frac{\partial MO^1(x)}{\partial x} |_{\beta=0} = b_I a_I \phi^1(x) \frac{\partial P(x)}{\partial x}. \hspace{1cm} (P9-5)$$

Since by definition $\frac{\partial P(x)}{\partial x} > 0$ for all $x \in [0, \bar{x})$ and $\frac{\partial P(x)}{\partial x} = 0$ for $x = \bar{x}$, it must be the case that $\frac{\partial MO^j(x)}{\partial x} > 0$ for all $x \in [0, \bar{x})$ and $\frac{\partial MO^1(x)}{\partial x} = 0$ for $x = \bar{x}$. Thus, the incumbent manager invests in all available positive NPV projects. Hence, the proof.

## 8.3 Proof of Proposition 10

Incumbent’s objective function if he finances the new investment issuing j-type shares, say $MO^j$, is

$$MO^j(x) = \beta NV_D^j(x) + \phi^j(x) b_I a_I P(x).$$  \hspace{1cm} (P10-1)

Differentiating the incumbent’s objective function with respect to $x$ gives

$$\frac{\partial MO^j(x)}{\partial x} = \beta N \frac{\partial V_D^j(x)}{\partial x} + b_I a_I \left( \frac{\partial \phi^j(x)}{\partial x} P(x) + \phi^j(x) \frac{\partial P(x)}{\partial x} \right). \hspace{1cm} (P10-2)$$

Because we are considering only positive NPV projects, increasing $x$ will always increase the expected dividend. Hence, $\frac{\partial V_D^j(x)}{\partial x} \geq 0$ for all $x \in [0, \bar{x}]$ and for all $j = 0, 1$. Also, we know that $\frac{\partial P(x)}{\partial x} |_{x=\bar{x}} = 0$. Hence, the sign of $\frac{\partial MO^j(x)}{\partial x} |_{x=\bar{x}}$ depends on the sign of $\frac{\partial \phi^j(x)}{\partial x} |_{x=\bar{x}}$. Thus, the sufficient condition for $\frac{\partial MO^j(x)}{\partial x} |_{x=\bar{x}} \geq 0$ is $\frac{\partial \phi^j(x)}{\partial x} |_{x=\bar{x}} \geq 0$.

From equation (23) we know that if voting shares are issued and $b_I < 1/2$, then $\frac{\partial \phi^1(x)}{\partial x} > 0$ for all $x$. Hence, the sufficient condition for the incumbent to invest $\bar{x}$ using voting shares is $b_I < 1/2$. Similarly, from equation (24) we know that if nonvoting shares are issued and $b_I \geq 1/2$, then $\frac{\partial \phi^0(x)}{\partial x} > 0$ for all $x$. Hence, the sufficient condition for the incumbent to invest $\bar{x}$ using nonvoting shares is $b_I \geq 1/2$. Hence, the proof.


8.4 Proof of Proposition 11

Differentiating equation P9-2, substituting \( j = 1 \) and further simplifying we obtain

\[
\frac{\partial MO^1(x)}{\partial x} = \beta N \frac{\partial V_1^I(x)}{\partial x} + b_I \alpha a_I \left( \frac{\partial \phi^1(x)}{\partial x} P(x) + \phi^1(x) \frac{\partial P(x)}{\partial x} \right).
\] (P11-1)

Let us consider following situations:

1. \( \beta = 0 \) and \( b_I \in [0, 1] \);
2. \( \beta > 0 \) and \( b_I \in [0, 1/2] \); and,
3. \( \beta > 0 \) and \( b_I \in [1/2, 1] \).

We have shown in Proposition 9 that irrespective of the incumbent manager’s private quality, the incumbent will fund all available positive NPV projects if \( \beta = 0 \). Hence, \( \beta > 0 \) is a necessary condition for under-investment. If \( \beta > 0 \), then \( \frac{\partial \phi^1(x)}{\partial x} < 0 \) if \( b_I \geq \frac{1}{2} \) and \( \frac{\partial \phi^1(x)}{\partial x} > 0 \) if \( b_I < \frac{1}{2} \).

Thus, if \( b_I < 1/2 \), then all three, \( \frac{\partial V_1^I(x)}{\partial x} \), \( \frac{\partial P(x)}{\partial x} \), and \( \frac{\partial \phi^1(x)}{\partial x} \) in equation P11-1, are increasing in all \( x \in [0, \bar{x}] \); hence, \( \frac{\partial MO^1(x)}{\partial x} \) is increasing in \( x \). If \( b_I \geq 1/2 \), then \( \frac{\partial V_1^I(x)}{\partial x} \) and \( \frac{\partial P(x)}{\partial x} \) are still increasing in \( x \), but \( \frac{\partial \phi^1(x)}{\partial x} \) is decreasing in \( x \). Thus, only way \( \frac{\partial MO^1(x)}{\partial x} \) can be negative is when \( \frac{\partial \phi^1(x)}{\partial x} < 0 \). But the only way \( \frac{\partial \phi^1(x)}{\partial x} < 0 \) is if \( \beta > 0 \) and \( b_I \geq 1/2 \). Hence, the proof.

8.5 Proof of Proposition 12

Using equation (BR-12) we find that the total value of voting shares,

\[
NV_1 = \frac{1}{2} P(x) \left( (a_I(1 - \alpha b_I) + (1 + \kappa^1) \left( 1 - \frac{\ln(1 + \alpha \kappa^1)}{\alpha \kappa^1} \right) \bar{a}_R \right.
\right) + (1 - \alpha/2) \left. (1 - (\bar{a}_R)^2) \right),
\] (P12-1)

and the total value of vote is

\[
NV_{vote}^1 = \frac{P(x)}{2} (1 + \kappa^1) \left( 1 - \frac{\ln(1 + \alpha \kappa^1)}{\alpha \kappa^1} \right) \bar{a}_R.
\] (P12-2)

Hence, the total value of the dividend claim is

\[
NV_D^1 = NV_1^1 - NV_{vote}^1 = \frac{1}{2} P(x) \left( (a_I(1 - \alpha b_I) + (1 - \alpha/2) (1 - (\bar{a}_R)^2)) \right).
\] (P12-3)

Thus, \( MO^1 \) can be expressed as

\[
MO^1 = \beta NV_D^1 + \phi^1 b_I \alpha a_I P(x) = P(x) \left( \frac{\beta}{2} \left( (a_I(1 - \alpha b_I) + (1 - \alpha/2) (1 - (\bar{a}_R)^2)) \right) + \phi^1 b_I \alpha a_I \right).
\] (P12-4)
Let \( A_{p11} = \frac{\beta}{2} ((a_I(1 - \alpha b_I) + (1 - \alpha/2) (1 - (\alpha b_I)^2))) + \phi^1 b_I \alpha a_I \). Differentiating equation (P12-4) with respect to \( x \) and rearranging terms, we have

\[
\frac{\partial M_0^1}{\partial x} = A_{p11}(x) \frac{\partial P(x)}{\partial x} + P(x) \frac{\partial A_{p11}(x)}{\partial x}.
\]  

From equation (P12-5) we know that \( \frac{\partial M_0^1}{\partial x} < 0 \) at \( x = \bar{x} \) implies that \( \frac{\partial A_{p11}(\bar{x})}{\partial x} < 0 \) because \( \frac{\partial P(\bar{x})}{\partial x} = 0 \) at \( x = \bar{x} \). Also, \( \frac{\partial A_{p11}(\bar{x})}{\partial x} < 0 \) when \( \frac{\partial A_{p11}(\bar{x})}{\partial x} < 0 \) implies that \( \frac{\partial A_{p11}(\bar{x})}{\partial x} > 0 \).

Differentiating \( A_{p11}(\bar{x}) \) with respect to \( \kappa_1^1 \) and rearranging terms, we have

\[
\frac{\partial A_{p11}(\kappa_1^1)}{\partial \kappa_1^1} = \frac{\alpha(1 + b_I \alpha \kappa_1^1)}{\kappa_1^1(1 + \alpha \kappa_1^1)} + \frac{b_I \alpha \log(1 + \alpha \kappa_1^1)}{\kappa_1^1} - \frac{(1 + b_I \alpha \kappa_1^1) \log(1 + \alpha \kappa_1^1)}{(\kappa_1^1)^2} - \left(1 - \frac{\alpha}{2}\right) \alpha \beta(1 + b_I \alpha \kappa_1^1).
\]  

For \( \frac{\partial A_{p11}(\kappa_1^1)}{\partial \kappa_1^1} > 0 \), we need

\[
\frac{\alpha(1 + b_I \alpha \kappa_1^1)}{\kappa_1^1(1 + \alpha \kappa_1^1)} + \frac{b_I \alpha \log(1 + \alpha \kappa_1^1)}{\kappa_1^1} > \frac{(1 + b_I \alpha \kappa_1^1) \log(1 + \alpha \kappa_1^1)}{(\kappa_1^1)^2} + \left(1 - \frac{\alpha}{2}\right) \alpha \beta(1 + b_I \alpha \kappa_1^1).
\]  

Solving for \( b_I \) such that the condition in equation (P12-7) is satisfied,

\[
b_I > \frac{2 \log(1 + \alpha \kappa_1^1)}{\kappa_1^1(2 + \alpha \kappa_1^1)} + \alpha (2 - \alpha) \beta - \frac{2 \alpha}{\alpha^2 \left(2 - \frac{\beta}{1 + \alpha} - \alpha (2 - \alpha) \beta \kappa_1^1\right)} - \frac{2 \alpha (1 - \alpha)^2}{\beta \left(1 - (1 - \alpha) \beta\right) \left(2 - \frac{\alpha (2 - \alpha)^2}{1 - \beta}\right)}.
\]  

We do not know the exact value of \( \kappa_1^1 \), but we do know the RHS of expression (P12-8) is strictly increasing in \( \kappa_1 \). Hence, we replace \( \kappa_1 \) by its maximum value \( \frac{\beta}{1 - \beta} \), and get the sufficient condition for under-investment as follows:

\[
b_I > \frac{\alpha (2 - \alpha) \beta + \frac{2 (1 - \beta)^2 \log(1 + \alpha \beta)}{\beta^2} - \frac{2 \alpha (1 - \alpha)^2}{\beta (1 - (1 - \alpha) \beta)}}{\alpha^2 \left(2 - \frac{\beta (2 - \alpha)^2}{1 - \beta}\right)} - \frac{2 \alpha (1 - \beta)^2}{\beta \left(1 - (1 - \alpha) \beta\right) \left(2 - \frac{\alpha (2 - \alpha)^2}{1 - \beta}\right)}.
\]  

### 8.6 Proof of Proposition 13

Consider the values of \( b_I \) for which the manager invests in all available positive NPV projects if nonvoting equity is used to finance the investment. Assume that the manager invests some \( x \) if voting equity is used to finance the investment. We have to obtain conditions such that \( V_1^0(\bar{x}) \geq V_1^1(x) \). Substituting for \( V_1^0(\bar{x}) \) and \( V_1^1(x) \) from equations (BR-12) and (BR-13), we
get

\[ V^0_1(\bar{x}) - V^1_1(x) = \frac{a_I(1 - \alpha b_I)}{2} (P(\bar{x}) - P(x)) + \frac{2 - \alpha}{4} (1 - a_I^2 - a_I^2 b_I \alpha \kappa^0(2 + b_I \alpha \kappa^0)) P(\bar{x}) \]

\[ - \frac{2 - \alpha}{4} (1 - a_I^2 - a_I^2 b_I \alpha \kappa^1(2 + b_I \alpha \kappa^1)) P(x) - \frac{P(x)}{2} (1 + \kappa^1) \left( 1 - \frac{\ln(1 + \alpha \kappa^1)}{\alpha \kappa^1} \right) \tilde{a}_R \]

\[ + \frac{P(\bar{x})}{2} \left( (1 + \kappa^0) - \frac{1}{1 - \beta} \right) \left( 1 - \frac{\ln(1 + \alpha \kappa^0)}{\alpha \kappa^0} \right) \tilde{a}_R \]

(P13-1)

But \( \tilde{a}_R = b_I \alpha a_I (\kappa^0 - \kappa^1) > 0 \) and \( P(\bar{x}) - P(x) > 0 \ \forall \text{x} \neq \bar{x} \). Thus, ignoring these terms we substitute maximum value of \( \kappa^1 \) and minimum value of \( \kappa^0 \), as both are strictly increasing in \( \kappa \), and simplifying we get

\[
\left( (1 + \kappa^0) - \frac{1}{1 - \beta} \right) \left( 1 - \frac{\ln(1 + \alpha \kappa^0)}{\alpha \kappa^0} \right) - (1 + \kappa^1) \left( 1 - \frac{\ln(1 + \alpha \kappa^1)}{\alpha \kappa^1} \right) \]

\[
= \frac{(1 - \beta) \log \left( \frac{\alpha \beta}{1 - \beta} + 1 \right) - \alpha \beta^2 - \beta(1 - \beta)^2 \log \left( \frac{\alpha}{1 - \beta} + 1 \right)}{\alpha(1 - \beta)\beta} > 0, \quad (P13-2)
\]

for all \( \alpha \in [0, 1] \) and for all \( \beta \in (0, 1/2) \). Using this result we can ignore these terms and rewrite equation P13-1 as

\[ V^0_1(\bar{x}) - V^1_1(x) = \frac{a_I(1 - \alpha b_I)}{2} (P(\bar{x}) - P(x)) + \frac{2 - \alpha}{4} (1 - a_I^2 - a_I^2 b_I \alpha \kappa^0(2 + b_I \alpha \kappa^0)) P(\bar{x}) \]

\[ - \frac{2 - \alpha}{4} (1 - a_I^2 - a_I^2 b_I \alpha \kappa^1(2 + b_I \alpha \kappa^1)) P(x). \quad (P13-3)
\]

After simplifying and rearranging we get

\[
1 - \frac{P(x)}{P(\bar{x})} \geq \frac{\frac{2 - \alpha}{4} ((1 - a_I^2 - a_I^2 b_I \alpha \kappa^1(2 + b_I \alpha \kappa^1)) - (1 - a_I^2 - a_I^2 b_I \alpha \kappa^0(2 + b_I \alpha \kappa^0)))}{\frac{2 - \alpha}{4} (1 - a_I^2 - a_I^2 b_I \alpha \kappa^1(2 + b_I \alpha \kappa^1)) + \frac{a_I(1 - \alpha b_I)}{2}}. \quad (P13-4)
\]

The expression, say \( J^1_{p11} = \frac{2 - \alpha}{4} (1 - a_I^2 - a_I^2 b_I \alpha \kappa^1(2 + b_I \alpha \kappa^1)) \), is a decreasing function of \( \kappa^1 \) and the entire expression is an increasing function of \( J^1_{p11} \). Thus, we substitute minimum value of \( \kappa^1 \) into the above expression. Similarly, the expression, say \( J^0_{p11} = \frac{2 - \alpha}{4} (1 - a_I^2 - a_I^2 b_I \alpha \kappa^0(2 + b_I \alpha \kappa^0)) \), is a decreasing function of \( \kappa^0 \), but the entire expression is a decreasing function of \( J^0_{p11} \). Thus, substituting maximum value of \( \kappa^0 = \frac{\beta}{1 - \beta} + \frac{1}{1 - \beta} \) and simplifying we get

\[
1 - \frac{P(x)}{P(\bar{x})} \geq \frac{a_I^2 \alpha b_I (2 - \alpha)(2 + b_I (2 \alpha \beta + \alpha) - 2 \beta)}{2 a_I (1 - \beta)^2 (1 - \alpha a_I) - (2 - \alpha)(a_I - a_I \beta(1 - \alpha b_I))^2 + (2 - \alpha)(1 - \beta)^2}. \quad (P13-5)
\]
8.7 Proof of Proposition 14

We prove this proposition in two parts: First we hold the investment level fixed and obtain conditions under which the manager is better off if the investment is financed using nonvoting shares; then, we show that the manager remains better off if the investment level is increased. From Proposition 11 we know that \(MO^j = A_{p_{1j}} P(x)\), where \(j\) stands for the type of shares issued. To show that \(MO^0 \geq MO^1\), we need to obtain conditions under which \(A(k^0) \geq A_{p_{11}}\).

Substituting for \(A_{p_{10}}\) and \(A_{p_{11}}\) from equations and simplifying we get

\[
\frac{\beta}{2} (a_I (1 - \alpha b_I) + (1 - \alpha/2)(1 - (\bar{a}_R^0)^2)) + \phi^0 b_I \alpha a_I > \frac{\beta}{2} ((1 - \alpha/2)(1 - (\bar{a}_R^0)^2)) + \phi^1 b_I \alpha a_I.
\]

(R14-1)

Rearranging the terms we get

\[
\phi^0 b_I \alpha a_I - \phi^1 b_I \alpha a_I > \frac{\beta}{2} ((1 - \alpha/2)(1 - (\bar{a}_R^0)^2)) - \frac{\beta}{2} ((1 - \alpha/2)(1 - (\bar{a}_R^0)^2)) .
\]

(R14-2)

After substituting for \(\bar{a}_R^j\) from equation (15) and \(\phi^j\) from equation (21) and on further simplification,

\[
4 \kappa^1 (1 + \alpha b_I \kappa^0) \log(1 + \alpha \kappa^0) - 4 \kappa^0 (1 + \alpha b_I \kappa^1) \log(1 + \alpha \kappa^1) - (2 - \alpha) \beta \kappa^0 \kappa^1 (\kappa^0 - \kappa^1) (2 + \alpha b_I (\kappa^0 + \kappa^1)) > 0.
\]

(P14-3)

Next, we solve for \(b_I\) such that the above inequality holds:

\[
b_I \geq \hat{b}_I = \frac{2(2 - \alpha)\alpha \beta(\kappa^0 - \kappa^1) + 4 \frac{1}{\kappa^0} \log(1 + \alpha \kappa^0) - 4 \frac{1}{\kappa^1} \log(1 + \alpha \kappa^1)}{\alpha(4 \log \frac{1 + \alpha \kappa^0}{1 + \alpha \kappa^0} + (2 - \alpha)\alpha \beta(\kappa^0 - \kappa^1)(\kappa^1 + \kappa^0))}.
\]

(P14-4)

As \(\kappa^1\) increases for \(\hat{b}_I\), so we substitute the maximum value of \(\kappa^1 = \frac{\beta}{1 - \beta}\) into the expression. But \(\hat{b}_I\) is also an increasing function of \(\kappa^0\) and unfortunately \(\kappa^0\) is unbounded:

\[
\kappa^0 = \frac{\beta N + n^0}{(1 - \beta)N} = 1 + \frac{n^0}{(1 - \beta)N}.
\]

(P14-5)

If we assume upper bound of \(n^0\) is \(N\), (i.e., if new shares issued are no greater than the existing number of shares), then \(\kappa^0 = 1 + \frac{1}{1 - \beta}\). If \(\beta = .1\), then \(\kappa^0 = 1 + \frac{1}{1 - .1} = 2.1111\) whereas if \(\beta = .4\), then \(\kappa^0 = 1 + \frac{1}{1 - .4} = 2.67\). Substituting maximum value of \(\kappa^1 = \frac{\beta}{1 - \beta}\) and maximum value of \(\kappa^0 = 1 + \frac{1}{1 - \beta}\) we get

\[
b_I \geq \hat{b}_I = \frac{2(2 - \beta) (\alpha(2 - \alpha) \beta^2 + (1 - \beta) \log \left(1 + \frac{\alpha \beta}{1 - \beta}\right)) - (1 - \beta) \beta \log \left(1 + \alpha + \frac{\alpha}{1 - \beta}\right)}{4 \alpha (2 - \beta) \beta \left(\log \left(1 + \alpha + \frac{\alpha}{1 - \beta}\right) - (1 - \beta) \log \left(1 + \frac{\alpha \beta}{1 - \beta}\right) - (2 - \alpha) \beta\right)}.
\]

(P14-6)
Equation P14-6 ensures that the manager is better off if the level of investment remains the same. The second part of the proof requires us to show that the manager is better off if the level of investment increases. This requires that $MO^{0}(x_2) > MO^{0}(x_1)$, where $x_2 > x_1$. But we know that if $b_I \geq \frac{1}{2}$, then $\frac{\partial MO^{0}(x)}{\partial x} > 0$. Hence, $\hat{b}_I \geq \frac{1}{2}$ is sufficient condition for the incumbent to undertake all available positive NPV projects. If $b_I \geq \hat{b}_I$, then the incumbent is better off if he is allowed to use nonvoting shares to fund new investment projects.

8.8 Proof of Proposition 15

Using equations (11) and (13) we get the incumbent’s minimum public quality required to win the control contest: If financed using voting shares, it is

$$a_I^1 \geq \frac{a_R (1 + \alpha \kappa^1 b_R)}{(1 + \alpha \kappa^1 b_I)}.$$  \hspace{1cm} (P15-1)

and if financed using nonvoting shares, it is

$$a_I^0 \geq \frac{a_R (1 + \alpha \kappa^0 b_R)}{(1 + \alpha \kappa^0 b_I)}.$$  \hspace{1cm} (P15-2)

We need to show that $a_I^0 \leq a_I^1$. Thus, we obtain

$$a_I^0 - a_I^1 = -\frac{a_R \alpha (b_I - b_R) (\kappa^0 - \kappa^1)}{(1 + \alpha \kappa^1 b_I) (1 + \alpha \kappa^0 b_I)} \leq 0.$$  \hspace{1cm} (P15-3)

From equation (BR-1) we know that $\kappa^0 - \kappa^1 > 0$. Thus, if $b_I \geq b_R$ then $a_I^0 < a_I^1$. Hence, the proof.