A Note on Information and the Cost of Capital in a Mean-Variance Efficient Market

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Abstract

Before information \( \phi \) arrives, it must be logically uncertain whether the stock price conditioned on \( \phi \) will be higher or lower than the current price. Any other inequality presents an obvious arbitrage opportunity. By assuming this minimal condition of efficient markets, it is shown under the mean-variance CAPM that information which makes the future value of a firm more certain, in the sense that its perceived covariance with the market is reduced towards zero, can lead to a higher expected return on that asset. A further connotation is that it is theoretically possible that the required return on the stock will necessarily fall after observing signal \( \phi \), or (in other circumstances) that it will necessarily rise. In general, information that allows better discrimination between firms leads some firms to have higher costs of capital and other firms to have lower costs of capital. Less obviously, better discrimination between firms can induce a higher average cost of capital across the market.

Keywords: Information, cost of capital, CAPM, efficient markets, asset pricing, financial disclosure, accounting information
1 Introduction

It is broadly accepted in accounting theory that improved financial statements (e.g. higher "quality" earnings measurement) should lead investors as a matter of logic to greater certainty, and therefore to a lower risk premium on capital. Better or more informative financial reporting is advocated as a mechanism by which firms can reduce the cost of capital. This hypothesis has been trumpeted by corporate regulators and appears frequently in both theoretical and empirical accounting research.

...high quality accounting standards...improve liquidity [and] reduce capital costs. (Levitt 1998, p.81)

More information always equates to less uncertainty, and ...people pay more for certainty. In the context of financial information, the end result is that better disclosure results in a lower cost of capital. (Foster 2003, p.1)

...we show that higher quality accounting information and financial disclosures affect the assessed covariances with other firms, and this effect unambiguously moves a firm’s cost of capital closer to the risk-free rate. (Lambert et al. 2007, p.387)

The intuition is straightforward. A firm’s cost of capital is the riskless interest rate plus a risk premium. Releasing more information and, in particular, more public information through financial reports and other public disclosures by firms reduces the uncertainty about the size and the timing of future cash flows and, therefore, also the risk premium. (Christensen et al. 2010 p.817)

The link between corporate disclosure, investor information and the cost of capital is one of the most fundamental relations in finance and accounting. Understanding this link is of substantial interest to firms that provide information to capital markets as well as to financial market regulators who mandate disclosures. Various theoretical models predict that an increase in information quality is negatively related to the cost of capital ...Similarly, the estimation risk literature suggests that higher quality information should manifest in lower systematic risk and expected returns. (Leuz and Schrand 2011)

The purpose of this note is to demonstrate that in an efficient market, additional or better information - which reduces investor uncertainty - can lead to an increase in the firm’s cost of capital, and must inevitably have that effect in some instances. This may not be evident intuitively and is important in the design of empirical studies of the link between information quality and the cost of capital. In an efficient market,
better disclosure will generally lead to some firms facing higher costs of capital, and can lead to a higher risk premium on average across the whole market. A further result is that it is possible that the required return on a given stock will necessarily fall upon receipt of new information, no matter whether the newly disclosed information is favorable or unfavorable. There are also circumstances under which the required return will necessarily increase after receiving more information.

2 A Higher Cost of Capital

Let $P_i$ be the current stock price of firm $i$. Assume that the revised price $P_i|\phi$ conditional on new information $\phi \in \{\phi^*, \phi^+\}$ is either $P_i^* = P_i|\phi^*$ or $P_i^+ = P_i|\phi^+$, where $P_i^* < P_i^+$. In an efficient market it is necessary that

$$P_i^* < P_i < P_i^+.$$ 

Any other inequality, such as for example $P_i < P_i^* < P_i^+$, would imply an obvious arbitrage opportunity. The only other plausible relationship, namely $P_i^* = P_i = P_i^+$, is ignored on the generalization that $\phi$ is not informative unless it causes a stock price change (note of course that this is not strictly correct since $\phi$ can in principle lead to changes in the joint probability distribution of future firm values that happen to compensate for one another in such a way that the price of firm $i$ stays the same).

The mean-variance CAPM expressed as a pricing equation is

$$P_i|\phi = \frac{1}{1 + r} \left\{ E[V_i|\phi] - \frac{\text{cov}(V_i, V_M|\phi)}{\text{var}(V_M)} \left( E[V_M] - P_M(1 + r) \right) \right\}, \quad (1)$$

where $V_i > 0$ is the terminal value of firm $i$, $V_M = (V_1 + V_2 + ...)$ is the corresponding market value, $P_M = (P_1 + P_2 + ...)$ is the aggregate price of the market, and $r$ is the risk free cash rate. See Fama (1976) and Fama and Miller (1972) on this form of the CAPM.

Given a positive market risk premium $(E[V_M] - P_M(1 + r))$, the CAPM equation (1) implies that $P_i|\phi$ is increasing in $E[V_i|\phi]$ and decreasing in $\text{cov}(V_i, V_M|\phi)$. It is assumed that the firm is a very small component of the market, and that the expected value of the market, its variance and its price are only negligibly affected by the firm-related information $\phi$. Hence, for the sake of analysis, $E[V_M], \text{var}(V_M)$ and $P_M$ are treated as constants, unaffected by $\phi$.

The notion $P_i|\phi$ of a stock price $P_i$ conditioned on information $\phi$ is consistent with either a direct-valuation or an input-to-valuation understanding of value-relevant information in accounting. Holthausen and Watts (2001) made it clear that the multiple roles of accounting information may weaken the information-theoretic (Bayesian) bond between firm value and accounting information $\phi$, however even where the primary role of $\phi$ stems more from a regulatory or contracting requirement, $\phi$ will typically retain incremental value-relevance in the sense that $P_i|\phi = P_i$ (or more strictly $P_i|\phi, \Omega = P_i|\Omega$, where $\Omega$ represents the set of all existing relevant information apart from $\phi$, since the marginal Bayesian effect of $\phi$ will hinge on what other information $\Omega$ is known at the time).

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For convenience in exposition, consider the case of \( V_i \geq 0 \) (for all firms \( i \)) and the typical firm \( i \) for which both unconditional and conditional covariances, \( \text{cov}(V_i, V_M) \) and \( \text{cov}(V_i, V_M|\phi) \), are positive. Also assume, following Lambert et al. (2007) and others, that information \( \phi \) resolves uncertainty in the sense that \( \text{var}(V_i|\phi) < \text{var}(V_i) \) and \( 0 < \text{cov}(V_i, V_M|\phi) < \text{cov}(V_i, V_M) \), for all possible signal observations \( \phi \). By this model, the unknown \( V_i \) becomes more certain (lower variance) the greater the information \( \phi \). In the limit, \( \phi \) is so revealing that \( V_i \) is a known non-negative constant and \( \text{cov}(V_i, V_M|\phi) = 0 \).

It follows from (1) that if \( 0 < \text{cov}(V_i, V_M|\phi^*) < \text{cov}(V_i, V_M) \), the "no arbitrage" market efficiency requirement that \( P_i^* = P_i|\phi^* < P_i \) is satisfied only if \( E[V_i|\phi^*] < E[V_i] \). Hence, in an efficient market, information \( \phi \) that brings greater certainty, such that \( 0 < \text{cov}(V_i, V_M|\phi) < \text{cov}(V_i, V_M) \), must sometimes (i.e. possibly) also bring a lower mean or expectation, \( E[V_i|\phi] < E[V_i] \).

It is sensible at this point to ensure that under the laws of probability these two conditions, namely \( \text{cov}(V_i, V_M|\phi) < \text{cov}(V_i, V_M) \) and \( E[V_i|\phi] < E[V_i] \), can occur together. If this is not possible, then there is a clash between market efficiency (no arbitrage) and the mean-variance CAPM.

The probability identity that connects covariance and expectation (mean) is

\[
\text{cov}(V_i, V_M|\cdot) = E[V_i V_M|\cdot] - E[V_i|\cdot]E[V_M].
\]  

(2)

This equation shows that while in general a change in the mean will be associated with a change in covariance, no such connection is certain, and as a rule the covariance \( \text{cov}(V_i, V_M|\cdot) \) can go up or down or stay the same under a given change in \( E[V_i|\cdot] \), and vice versa. It is possible therefore, consistent with intuition, that information \( \phi \) can lead jointly to a decrease in the covariance (towards zero) and a decrease in the expectation. This occurs when \( \phi \) brings a decrease in \( E[V_i|\cdot] \) sufficiently large to offset the decrease in \( E[V_i|\cdot]E[V_M] \) that comes as a result of a lower expectation \( E[V_i|\cdot] \) (assuming non-negative \( V_i \) and \( V_M \)).

The final step is to show that information \( \phi \) prompting simultaneous decreases in covariance and mean, while obeying (2), can have the net effect of increasing the CAPM required return. From (1) the required return on the firm is

\[
E[R_i|\phi] = \frac{E[V_i|\phi]}{P_i|\phi} = \frac{E[V_i|\phi](1 + r)}{E[V_i|\phi] - \frac{\text{cov}(V_i, V_M|\phi)}{\text{var}(V_M)}(E[V_M] - P_M(1 + r))}.
\]  

(3)

By definition, the arrival of information \( \phi \) brings about an increase in the cost of capital if \( E[R_i|\phi] > E[R_i] \). From (3), and taking the realistic case where both covariances \( \text{cov}(V_i, V_M) \) and \( \text{cov}(V_i, V_M|\phi) \) are positive, \( E[R_i|\phi] > E[R_i] \) occurs if

\[
\frac{E[V_i|\phi]}{E[V_i]} < \frac{\text{cov}(V_i, V_M|\phi)}{\text{cov}(V_i, V_M)}.
\]  

(4)

Thus, if information \( \phi \) reduces the assessed covariance by some fixed percentage, then the cost of capital will increase whenever that same information \( \phi \) reduces the mean.
by more than the same percentage (i.e. a 10% reduction in the covariance leads to an increase in the cost of capital if the posterior mean is more than 10% lower than the prior mean); cf. Lambert et al. (2007).

Imposing the identity (2), this condition can be re-written very neatly in terms of expectations

\[ \frac{E[V_i|\phi]}{E[V_i]} < \frac{E(V_i, V_M|\phi)}{E(V_i, V_M)}. \]  

(5)

3 An Efficient Market Paradox

The preceding analysis reveals how information that adds to investor certainty can also add to the investor’s required return on capital. An interesting finding is not merely that this can happen, and realistically will happen frequently, but that in an efficient market it does not necessarily have to happen ever.

The "no arbitrage" precondition \( P_i^* < P_i < P_i^+ \) requires that the asset price can rise or fall upon observation of signal \( \phi \), implying that the investor can win or lose. It is logically possible, however, even when that observation necessarily adds to certainty, in the sense that \( 0 < \text{cov}(V_i, V_M|\phi) < \text{cov}(V_i, V_M) \), that both conditional expected returns are less than the prior expected return, that is

\[ E[R_i|\phi^+] < E[R_i] \quad \text{and} \quad E[R_i|\phi^+] < E[R_i]. \]

This occurs when condition (4) or equivalently (5) is not met for either of the potential values of signal \( \phi \in \{\phi^*, \phi^+\} \).

First take the case of \( \phi = \phi^+ \) (\( \phi \) is favorable), upon which it is required that \( P_i < P_i^+ \). A signal \( \phi^+ \) can lead jointly to a reduction in covariance and \( P_i < P_i^+ \) under either an increase or (sufficiently small) decrease in mean. Note the technical point here that a signal \( \phi^+ \) that is "favorable" in the sense that \( P_i^+ > P_i \) need not bring about an increased mean, \( E[V_i|\phi^+] > E[V_i|\phi^+] \). Rather, \( \phi^+ \) can produce a higher stock price when \( E[V_i|\phi^+] \) is somewhat lower than \( E[V_i] \) merely by informing the market that the stock is sufficiently less risky (in a covariance sense) relative to previous perceptions.

It can happen, therefore, that the favorable signal \( \phi^+ \) will result in (i) a lower covariance \( 0 < \text{cov}(V_i, V_M|\phi^+) < \text{cov}(V_i, V_M) \), and (ii) a higher price \( P_i^+ > P_i \), and (iii) either a higher or lower mean \( E[V_i|\phi^+] \). In this situation, the cost of capital must necessarily fall upon observing \( \phi^+ \), whether the mean goes up or down. Specifically, if \( E[V_i|\phi^+] > E[V_i] \), then by (3) the conditional required return must be lower because both the reduced covariance and increased mean push in that direction. And if \( E[V_i|\phi^+] \leq E[V_i] \), then \( P_i^+ = E[V_i|\phi^+]/E[R_i|\phi^+] \) is greater than \( P_i = E[V_i]/E[R_i] \) only if \( E[R_i|\phi^+] \) is less than \( E[R_i] \), by some sufficient margin. So the cost of capital can only be lower after observing \( \phi^+ \).

Now consider the case of \( \phi = \phi^* \) (\( \phi \) is unfavorable), upon which it is required that \( P_i^* < P_i \). It was found above that, when \( 0 < \text{cov}(V_i, V_M|\phi^*) < \text{cov}(V_i, V_M) \), the
condition \( P_i^* < P_i \) implies \( E[V_i|\phi^*] < E[V_i] \). More particularly, from (1) this pricing condition can be written as

\[
\frac{E[V_i|\phi^*]}{E[V_i]} < 1 - \frac{k(\text{cov}(V_i, V_M) - \text{cov}(V_i, V_M|\phi^*))}{E[V_i]} < 1, \tag{6}
\]

where \( k = (E[V_M] - P_M (1 + r)) / \text{var}(V_M) \) is the market "price of risk". A signal \( \phi^* \) that meets this condition leads to a lower cost of capital, \( E[R_i|\phi^*] < E[R_i] \), if by the opposite of (4)

\[
\frac{E[V_i|\phi^*]}{E[V_i]} > \frac{\text{cov}(V_i, V_M|\phi^*)}{\text{cov}(V_M)} . \tag{7}
\]

It follows that \( \phi = \phi^* \) can satisfy both (6) and (7) only if

\[
\frac{\text{cov}(V_i, V_M|\phi^*)}{\text{cov}(V_i, V_M)} < 1 - \frac{k(\text{cov}(V_i, V_M) - \text{cov}(V_i, V_M|\phi^*))}{E[V_i]},
\]

for which it is sufficient that

\[
k < \frac{E[V_i]}{\text{cov}(V_i, V_M)} .
\]

Thus, provided that the market price of risk \( k \) does not exceed this positive (and plausible looking) bound, both (6) and (7) can be satisfied jointly, implying that signal \( \phi^* \) can bring a lower cost of capital while at the same time leaving \( P_i^* < P_i \).

It is theoretically conceivable therefore that \( E[R_i|\phi] < E[R_i] \) for both \( \phi = \phi^* \) and \( \phi = \phi^+ \). In this somewhat paradoxical situation, the investor foresees that the expected return on the stock will be lower after observing \( \phi \in \{\phi^*, \phi^+\} \) than it is beforehand. This does not imply however that there is any arbitrage opportunity or reason to sell immediately. The fact remains that the stock price will either go up to \( P_i^+ \) or down to \( P_i^* \) when conditioned on \( \phi \in \{\phi^*, \phi^+\} \), and the investor will hold the stock if she views this as a worthy gamble. Afterwards, she will have either a higher market value \( P_i^+ > P_i \) invested at a lower rate, or a lower market value \( P_i^* < P_i \) invested at a lower rate. That is the gamble offered by the market.

An investor may well take up this gamble. All that it entails is a wager of net amount \( (P_i - P_i^*) > 0 \) to win net amount \( (P_i^+ - P_i) > 0 \). Such a bet is rational for a risk averse investor provided that the subjective probability of winning (i.e. a price increase from \( P_i \) to \( P_i^+ \)) exceeds the market-implied risk-neutral probability of \( (P_i (1 + r) - P_i^*)/(P_i^+ - P_i^*) \) by sufficient margin to overcome that investor’s risk aversion (\( r \) is the risk free interest rate).

A further twist which follows easily from the argument above is that although \( \phi = \phi^+ \) leads to a higher stock price than \( \phi = \phi^* \), the revised expected return on the stock can be higher when \( \phi \) is favorable (\( \phi = \phi^+ \)) than when it is unfavorable (\( \phi = \phi^* \)). This is a quirk of the separate effects on stock price of covariance and expectation and of the way that different signals \( \phi^* \) and \( \phi^+ \) can have relatively disparate effects on the covariance and expectation.
In the atypical situation where cov$(V_i, V_M|\phi) < \text{cov}(V_i, V_M)$ can bring a higher required rate of return for both $\phi = \phi^*$ and $\phi = \phi^+$. It was found above that in this case $E[R_i|\phi^+]$ can only be lower than $E[R_i]$ whatever the effect of $\phi^+$ on the mean. This can be proved by contradiction. Specifically, if $E[R_i|\phi^+] > E[R_i]$, then $P^+_i = E[V_i|\phi^+]/E[R_i|\phi^+]$ can be greater than $P_i = E[V_i]/E[R_i]$ only if $E[V_i|\phi^+] > E[V_i]$, but in that case the cost of capital must go down rather than up, since a lower covariance and higher mean both push the cost of capital downwards. So there is a logical inconsistency in the proposition that $\phi^+$ can lead to a higher cost of capital.

### 3.1 Allowing for Negative Covariances

In the atypical situation where cov$(V_i, V_M) < \text{cov}(V_i, V_M|\phi) < 0$, greater certainty is represented by a higher covariance, and the expected return $E[R_i|\phi]$ given by (3) is increasing rather than decreasing in $E[V_i|\phi]$. The effect of this reversal is that it becomes theoretically conceivable that $E[R_i|\phi] > E[R_i]$ for both $\phi = \phi^*$ and $\phi = \phi^+$, while it remains feasible that $E[R_i|\phi] < E[R_i]$ for both $\phi = \phi^*$ and $\phi = \phi^+$.

Consider the case of $\phi = \phi^+$ (the signal is favorable). It follows from (1) that when there is an increase in covariance, $\text{cov}(V_i, V_M|\phi^+) > \text{cov}(V_i, V_M)$, the joint conditions of $E[R_i|\phi^+] > E[R_i]$ and $P^+_i > P_i$ are met if

$$\frac{E[V_i|\phi^+]}{E[V_i]} > \frac{\text{cov}(V_i, V_M|\phi^+)}{\text{cov}(V_i, V_M)} < 1,$$

and

$$\frac{E[V_i|\phi^+]}{E[V_i]} > 1 + \frac{k(\text{cov}(V_i, V_M|\phi^+ - \text{cov}(V_i, V_M))}{E[V_i]} > 1. \quad (8)$$

Given that $\text{cov}(V_i, V_M) < \text{cov}(V_i, V_M|\phi^+) < 0$, both of these bounds are positive and the effective constraint is (8). All that is required therefore is that signal $\phi^+$ brings a sufficiently large increase in mean (relative to the associated increase in covariance).

Now consider the case of $\phi = \phi^*$ (the signal is unfavorable), on which $\text{cov}(V_i, V_M|\phi^*) > \text{cov}(V_i, V_M)$. The joint conditions of $E[R_i|\phi^*] > E[R_i]$ and $P^*_i < P_i$ are met if

$$\frac{\text{cov}(V_i, V_M|\phi^*)}{\text{cov}(V_i, V_M)} < \frac{E[V_i|\phi^*]}{E[V_i]} < 1 + \frac{k(\text{cov}(V_i, V_M|\phi^*) - \text{cov}(V_i, V_M))}{E[V_i]}$$

This is feasible if

$$\frac{\text{cov}(V_i, V_M|\phi^*)}{\text{cov}(V_i, V_M)} < 1 + \frac{k(\text{cov}(V_i, V_M|\phi^*) - \text{cov}(V_i, V_M))}{E[V_i]}$$

for which it is sufficient that the price of risk $k > E[V_i]/\text{cov}(V_i, V_M)$.

This analysis reveals that by constraining the possible information events $\phi^*$ and $\phi^+$ in terms of their effects on mean and covariance, it becomes possible that the conditional expected return $E[R_i|\phi]$ is bound to exceed the unconditional expected
return $E[R_i]$ no matter whether the observed signal is $\phi = \phi^*$ or $\phi = \phi^+$ (while holding $P_i^* < P_i < P_i^+$). In these restricted yet plausible circumstances, the investor knows ex ante that whatever signal $\phi \in \{\phi^*, \phi^+\}$ eventuates, the revised covariance will increase towards zero while remaining negative, and that the effect of that observation $\phi$ on the expected value of $V_i$ will be such that the revised cost of capital must increase. This possibility represents a stark counter-example to the theoretical precept that more information brings a lower cost of capital. Again, of course, there is no arbitrage possibility, for the reasons as explained above.

4 Information and Certainty

In the analysis above, it is assumed that the signal $\phi \in \{\phi^*, \phi^+\}$ always makes the uncertain asset value $V_i$ more certain, no matter whether the observed signal is $\phi^+$ (favorable) or $\phi^*$ (unfavorable). Indeed, there are quite standard Bayesian probability problems in which even the smallest quantity of data or sample size is guaranteed to make the posterior distribution of the unknown quantity more peaked (lower dispersion or variance) than the prior distribution. Lambert et al. (2007, pp.395-6) invoke just such a model.

To avoid being misled, however, by the postulate that the arrival of new or more information necessarily resolves some amount of uncertainty, it should be noted that there are many realistic contexts and models in which this is not so. Elementary probability problems can be devised to prove as much. For example, suppose that there are two urns, one containing 50% red chips and the other containing 90% red chips. The probability of a random draw from a random urn producing a red chip is thus 70%, which is a relatively certain belief. Now suppose that we learn with probability one that the urn from which the chip came was the one with 50% red chips. We are thus returned to a state of maximum possible uncertainty (i.e. maximum variance or entropy). This simple example reveals that highly relevant and valuable information does not necessarily make the unknown more certain.

If certainty or risk is measured by variance, the general relationship between information and certainty is contained in the law of conditional variance

$$\text{var}(V_i) = E[\text{var}(V_i|\phi)] + \text{var}(E[V_i|\phi]).$$

This identity implies that the average variance after observing information $\phi$ is lower than the prior variance, that is $E[\text{var}(V_i|\phi)] < \text{var}(V_i)$, but not that the posterior variance is always lower. It follows therefore that even in the subclass of models for which certainty is fully captured by the variance of the posterior distribution, more data does not necessarily leave more certainty.

Similarly, and most importantly for mean-variance portfolio theory, consider the law of conditional covariance

$$\text{cov}(V_i, V_M) = E[\text{cov}(V_i, V_M)|\phi] + \text{cov}(E[V_i|\phi], E[V_M|\phi]).$$
Since the covariance of the conditional expectations $\text{cov}(E[V_i|\phi], E[V_M|\phi])$ can be negative while the expected covariance $E[\text{cov}(V_i, V_M)|\phi]$ is positive, it follows from this identity that the expected conditional covariance over the possible information events $\phi$ need not be lower than the prior covariance. So the CAPM measure of risk or uncertainty, that is $\text{cov}(V_i, V_M|\cdot)$, need not tend towards zero upon receipt of information $\phi$ (unless of course that information is perfect in the sense that $V_i$ is known with zero variance).

Importantly, it should be noted that information that adds to uncertainty remains welcome, as does any information relating to $V_i$ or $V_M$. The investor’s probabilities are on the whole more accurate when based on more information, and more accurate probabilities bring greater expected payoffs and utility (both ex ante and ex post). In the simple case of the urns, the decision maker is obviously better off knowing that the chip did not come from the 90% urn. This news may be disappointing and of course leaves greater uncertainty, but nonetheless helps to prevent overinvestment based on false confidence. See Blume and Easley (2006) and Sandroni (2000) on the related topic of "economic Darwinism", which underpins efficient markets. Similarly, see MacLean et al. (1992; 2004) on the typically much greater cost or lost utility of overinvestment compared to underinvestment.

The moral of this example for accounting theory is that earnings measures and other financial disclosures need not make the market more certain to be of great value to market participants. To the contrary, accounting information might be evaluated on how successfully its quells overconfidence, as much as by how widely - or moreover how appositely - it bolsters confidence. In those typical business and investment problems exhibiting much "natural" uncertainty, investors will not be assisted on average by information that suggests otherwise.

The second point is that although it is logically conceivable that all possible signal observations make things more certain, it is illogical to imagine that all possible signal outcomes will make things less certain. By Bayes theorem, the current probability of a given event is the expected future probability of that event. So if the possible signals are $\phi^*$ and $\phi^+$, and the respective Bayesian posterior probabilities of the event in question are $p^*$ and $p^+$, then the current probability (being an expectation or average of the two) must lie between $p^*$ and $p^+$, in which case at least one of the two possible future probabilities $p^*$ and $p^+$ must be closer to 0 or 1 (certainty) than the current probability.

5 Pricing Relevant Subsets

It seems unlikely that better information that allows better discrimination between firms in the market can reduce the cost of capital for all firms. A more natural presumption is that the market can benefit from better financial disclosure in the same way as the life insurance industry uses better medical tests to identify relevant subsets of the human population. By partitioning the population into statistically
relevant subsets, some individuals are seen as materially higher or lower than average risk, and are priced accordingly. This is akin to the "lemons" example of Healy and Palepu (2001) where better information allows the market to discriminate in price between good ideas and bad ideas.

The following simple but insightful model of mean-variance asset pricing reveals that better discrimination between firms in the market can either increase or decrease the cost of capital, not only for individual firms or subsets of firms but also on average across the whole market.

Suppose for the sake of simplicity that the market contains just two risky assets, \( i = 1, 2 \). Both risky assets are binaries with uncertain payoff \( V_i \in \{0, 1\} \). A risk averse investor with quadratic utility (so as to stay with mean-variance) must allocate her wealth \( W \) between the two risky assets and the risk free asset. This investor can be considered as the "representative" investor or more simply as the only investor in the market. Her utility function for wealth \( W \) is \( U(W) = W - \frac{1}{2} b W^2 \) (\( b > 0 \)). Marginal utility \( U'(W) = \frac{1}{b} W \) is positive for \( W < \frac{1}{b} \).

The investor endowed with initial wealth \( W = 1 \) maximizes expected utility

\[
E \left[ \left( w_1 R_1 + w_2 R_2 + (1 - w_1 - w_2) R_f \right) - \frac{b}{2} \left( w_1 R_1 + w_2 R_2 + (1 - w_1 - w_2) R_f \right)^2 \right],
\]

by selecting the best possible risky asset weights \( w_1 = w_1^* \) and \( w_2 = w_2^* \), subject to given asset prices \( P_1 \) and \( P_2 \), where \( R_1 = V_1 / P_1, R_2 = V_2 / P_2 \) and \( R_f = (1 + r) \) are the associated asset returns (expressed as factors).

In the calculations below, it is assumed that \( R_f = 1.10 \) and \( b = 1/3 \). The risk-free return \( R_f \) is chosen arbitrarily but the choice of \( b \) is governed by the need to ensure that the investor’s quadratic utility function \( U(W) \) is increasing over all feasible \( W \).

Differentiating expected utility with respect to weight \( w_i \) gives the first order condition

\[
E \left[ (R_i - R_f) - b(R_i - R_f) R_{opt} \right] = 0,
\]

where \( R_{opt} = (w_1 R_1 + w_2 R_2 + (1 - w_1 - w_2) R_f) \) is the weighted average return on the investor’s optimal overall portfolio of risky and risk-free assets. Taking note of the identity \( \text{cov}(R_i - R_f, R_{opt}) = E[(R_i - R_f) R_{opt}] = E[R_i - R_f] E[R_{opt}] \), the first order condition simplifies to a more familiar looking pricing equation

\[
E[R_i] = R_f + \frac{b \text{cov}[R_i, R_{opt}]}{1 - b E[R_{opt}]}.
\] (9)

This asset pricing model can now be used to find equilibrium asset prices \( P_1 \) and \( P_2 \), and hence the price \( (P_1 + P_2) \) of the "market portfolio" of risky assets and lastly the expected return on that risky asset portfolio, \( E[R_M] = E[V_M] / P_M = E[V_1 + V_2] / (P_1 + P_2) \). Note that the investor has wealth \( W = 1 \) so the amount invested in risk free assets is \( 1 - P_1 - P_2 \).
By definition, the equilibrium values of \( P_1 \) and \( P_2 \) are those prices satisfying (6) such that the investor’s optimal risky portfolio weights \( W^*_1 = w^*_1/(w^*_1 + w^*_2) \) and \( W^*_2 = w^*_2/(w^*_1 + w^*_2) \) are the market portfolio weights. That is
\[
W^*_1 = \frac{P_1}{(P_1 + P_2)} \quad \text{and} \quad W^*_2 = \frac{P_2}{(P_1 + P_2)}.
\]
For this illustration, asset prices are calculated separately under four different sets of information, \( \Phi_1, \Phi_2, \Phi_3, \Phi_4 \). The purpose of these calculations is to exhibit how new information can drive the cost of capital up or down, both for individual firms and for the market as a whole.

**Information \( \Phi_1 \).** Under the initial and rather primitive information set \( \Phi_1 \), the two assets are perceived as independent and as having the same probability of success, \( p(V_1 = 1) = p(V_2 = 1) = 0.6 \) (for convenience, the assumed information \( \Phi \) is omitted from the notation). The resulting asset prices under \( \Phi_1 \), found numerically so as to satisfy equilibrium conditions (9) and (10), are \( P_1 = P_2 = 0.409 \). These prices imply expected returns \( E[R_1] = E[V_1]/P_1 = 0.6/0.409 = 1.47 \), \( E[R_2] = E[R_1] = 1.47 \), and \( E[R_{opt}] = P_1R_1 + P_2R_2 + (1 - P_1 - P_2)R_f = 1.40 (R_f = 1.10) \).

The investor’s expected return on the overall optimal investment in risky and risk free assets is thus a factor of 1.40 or in the usual terms a return of 40%. The corresponding expected return on the "market portfolio" containing only the two risky assets is \( E[R_M] = E[V_1 + V_2]/(P_1 + P_2) = 1.47 \), or 47%. The implied market risk premium under risk aversion \( b = 1/3 \) is therefore \( 1.47 - 1.10 = 37\% \).

Before proceeding to revise the two asset prices under new information, it is reassuring to verify that the prices calculated using the expected quadratic utility pricing rule (9), which incorporates the investor’s risk aversion parameter \( b \) into the market risk premium, are mathematically consistent with the conventional CAPM formula (3). Briefly, the associated calculations are as follows:

\[
E[V_M] = E[V_1 + V_2] = 1.2
\]
\[
P_M = P_1 + P_2 = 0.818
\]
\[
\text{cov}(V_1, V_M) = \text{var}(V_1) + \text{cov}(V_1, V_2)
\]
\[
= 0.24 + 0 = 0.24
\]
\[
\text{var}(V_M) = \text{var}(V_1) + \text{var}(V_2) + 2\text{cov}(V_1, V_2)
\]
\[
= 0.24 + 0.24 + 0 = 0.48
\]
and hence by (3)
\[
E[R_1] = \frac{E[V_1]R_f}{E[V_1] - \frac{\text{cov}(V_1,V_M)}{\text{var}(V_M)} (E[V_M] - P_M R_f)} = 1.47.
\]
Similarly, by (3), \( E[R_2] = 1.47 \). The two asset prices \( P_1 = 0.409 \) and \( P_2 = 0.409 \) are thus consistent with the usual CAPM equation. This is the case also for the prices and returns calculated under the three further information sets described below.
**Information \( \Phi_2 \).** Under information \( \Phi_2 \), the two assets are identical but dependent. There is an underlying economic condition which can be either \( G \) (good) or \( B \) (bad), where \( \Pr(G) = \Pr(B) = 0.5 \). The relevant probabilities are the same for both risky assets. Specifically, \( \Pr(V_i = 1|G) = 0.85 \) and \( \Pr(V_i = 1|B) = 0.35 \) \((i = 1, 2)\). The unconditional probability is thus

\[
\Pr(V_i = 1) = \Pr(G) \Pr(V_i = 1|G) + \Pr(B) \Pr(V_i = 1|B) = 0.6, \quad (i = 1, 2)
\]

consistent with \( \Phi_1 \). An important difference between \( \Phi_1 \) and \( \Phi_2 \) is that the two risky assets are perceived under \( \Phi_2 \) as dependent and having positive covariance.

**Information \( \Phi_3 \).** Information set \( \Phi_3 \) represents a further refinement on \( \Phi_2 \). The revised conditional probabilities are now different for the two assets. Specifically,

\[
\begin{align*}
\Pr(V_1 = 1|G) &= 0.9 & \Pr(V_2 = 1|G) &= 0.8 \\
\Pr(V_1 = 1|B) &= 0.2 & \Pr(V_2 = 1|B) &= 0.5.
\end{align*}
\]

Note that these more deeply conditioned probabilities are still consistent with the less conditioned probabilities in \( \Phi_2 \), and hence also \( \Phi_1 \). For a randomly selected risky asset \( i \), \( \Pr(V_i = 1|G) = 0.85 \) and \( \Pr(V_i = 1|B) = 0.35 \), which match the conditional probabilities for both assets under \( \Phi_2 \). In effect, therefore, the average (i.e. unconditional) population probability \( \Pr(V = 1) = 0.6 \) is unchanged from information \( \Phi_1 \). However there are now two levels of conditioning or stratification within the population. The relevant conditioning factors are (i) whether the economy is \( G \) or \( B \) and (ii) whether the asset is of type \( i = 1 \) or type \( i = 2 \).

**Information \( \Phi_4 \).** Information set \( \Phi_4 \) is a direct refinement on \( \Phi_1 \), and is thus an alternative to \( \Phi_3 \), since both are feasible refinements of \( \Phi_1 \). The revised conditional probabilities are:

\[
\begin{align*}
\Pr(V_1 = 1|G) &= 0.7 & \Pr(V_2 = 1|G) &= 0.5 \\
\Pr(V_1 = 1|B) &= 0.4 & \Pr(V_2 = 1|B) &= 0.8.
\end{align*}
\]

Equilibrium prices and other related results calculated under the four different information levels are listed in Table 1.

First consider the required returns of the individual firms. Under \( \Phi_2 \), \( E[R_1] \) and \( E[R_2] \) increase relative to \( \Phi_1 \), essentially because the two assets are now viewed as dependent and hence not so attractive in portfolio. Under \( \Phi_3 \), which represents a further improvement in the sense that \( \Phi_3 \supset \Phi_2 \supset \Phi_1 \), where \( \supset \) represents a weak subset relationship, a distinction is made between the two assets and \( E[R_1] \) increases while \( E[R_2] \) decreases, relative to their values under \( \Phi_2 \). This is a clear example of how better information, that allows better discrimination between statistical subsets of assets, can help some assets and hurt others.

Similarly, consider information \( \Phi_4 \) which like \( \Phi_3 \) could have arisen directly from \( \Phi_1 \). Under \( \Phi_4 \), \( E[R_2] \) is lower than under \( \Phi_1 \) and \( E[R_1] \) is unchanged, whereas under
both rates of return are appreciably higher than under $\Phi_1$. The possible effects of information on firm returns are thus quite unpredictable. When firms provide information that allows the market to discriminate better between them, such as perhaps when they are all subject to more stringent mandatory financial reporting standards, it is likely that some firms will be left with a much increased cost of capital.

**Table 1**

<table>
<thead>
<tr>
<th>Information Set</th>
<th>$\Phi_1$</th>
<th>$\Phi_2$</th>
<th>$\Phi_3$</th>
<th>$\Phi_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[V_1]$</td>
<td>0.6</td>
<td>0.6</td>
<td>0.55</td>
<td>0.55</td>
</tr>
<tr>
<td>$E[V_2]$</td>
<td>0.6</td>
<td>0.6</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>$P_1$</td>
<td>0.409</td>
<td>0.362</td>
<td>0.320</td>
<td>0.375</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.409</td>
<td>0.362</td>
<td>0.423</td>
<td>0.477</td>
</tr>
<tr>
<td>$E[R_1]$</td>
<td>1.467</td>
<td>1.657</td>
<td>1.718</td>
<td>1.467</td>
</tr>
<tr>
<td>$E[R_2]$</td>
<td>1.467</td>
<td>1.657</td>
<td>1.537</td>
<td>1.363</td>
</tr>
<tr>
<td>$E[R_{opt}]$</td>
<td>1.400</td>
<td>1.504</td>
<td>1.483</td>
<td>1.363</td>
</tr>
<tr>
<td>$E[R_M]$</td>
<td>1.467</td>
<td>1.657</td>
<td>1.615</td>
<td>1.408</td>
</tr>
</tbody>
</table>

The more striking result is that the market risk premium can itself increase when there is better information about the firms that make up the market. The market risk premium $E[R_M] - R_f$ is higher under both $\Phi_2$ and $\Phi_3$ than it is under the most rudimentary information $\Phi_1$, so poor information can sometimes cause an efficient market to under-charge rather than over-charge for capital. On the other hand, the advent of more detailed information can sometimes assist firms to raise capital at lower cost, since the market risk premium $E[R_M] - R_f$ is lower under information $\Phi_4$ than under $\Phi_1$.

Note of course that a firm faced with a lower required return on capital, brought on by the disclosure of a new level of information, may not see that it has benefitted from that disclosure. If the improved information led to both a lower cost of capital and a lower share price, managers will likely be penalized for failing in their aim of maximizing the value of the firm.

6 Conclusion

Information may often reduce uncertainty, but this expectation does not imply that more rigorous disclosure of accounting information about the firm will induce the market or individual investors to require a lower risk premium on the firm’s stock. To the contrary, it is possible that new or better information can lead to both greater certainty (less diffuse probability beliefs) and a higher required return on capital. This proposition is demonstrated within a mean-variance CAPM under the elementary
market efficiency requirement that before observing any given signal \( \phi \) the market must be unsure whether the revised stock price conditioned on \( \phi \) will be higher or lower than the current price.

A further result is that information - which by definition changes the investor’s personal probabilities - can lead necessarily to greater certainty and a lower cost of capital, or to greater certainty and a higher cost of capital, but cannot lead necessarily to reduced certainty. That is not to say however that it cannot sometimes, frequently, or indeed helpfully, lead to both reduced certainty and a higher cost of capital. The final result shown in this paper is that when information allows better discrimination between firms, the market average cost of capital can go up rather than down. More information is thus potentially detrimental to both the firm’s cost of capital and the market risk premium.

It is conceivable practically as well as theoretically that improved information (e.g. more rigorous accounting earnings measurement) can increase the market risk premium. This might occur for example if the previous disclosure regime encouraged market participants to generally over-estimate expected cash flows or under-estimate their (generally positive) covariances. Conversely, if the previous reporting regime led to the opposite biases, perhaps through conservatism for example, then improved reporting can reduce the market wide cost of capital.

An interesting implication for firms that seek to reduce their cost of capital by better disclosure is that the more that they distinguish their activities from those of other firms in the market, or the more that they demonstrate that their earnings streams and associated risks are firm-specific or idiosyncratic, the more that the market will reward that disclosure. Traditionally, accounting information has a role in assisting users to estimate expected cash flows. The less obvious proposition is that accounting information should also assist users to assess the firm’s relationship with the market and its inherent dependence or otherwise on the macro-level drivers of costs and revenues that it cannot avoid. Put another way, part of the "value relevance" role of accounting information (cf. Bath et al. 2001) is to assist the market not only to predict future cash flows, but also to assess their covariance with the market and thus the cost of capital applicable to such cash flows.

References


